THE WALSH-SUZZALLO ARITHMETICS

EIGHTH YEAR BOOK

WALSH-SUZZALLO ARITHMETICS

BY

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By Grades

EIGHTH YEAR

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THE WALSH-SUZZALLO ARITHMETICS

Three-Book Course

I. Fundamental Processes
II. Essentials
III. Business and Industrial Practice

Two-Book Course

I. Fundamental Processes
II. Practical Applications

Course by Grades

I. Third Year Arithmetic
II. Fourth Year Arithmetic
III. Fifth Year Arithmetic
IV. Sixth Year Arithmetic
V. Seventh Year Arithmetic
VI. Eighth Year Arithmetic

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PREFACE

THESE texts in arithmetic are devised to train children to meet the ordinary demands of life. Practical efficiency, rather than mental discipline, has been the chief aim in their organization. Whatever the average man needs to know in mathematical terms, because of its frequency of occurrence in his life or because of its urgency and importance, has been here included.

This volume gives drill and review of the essentials covered in the preceding books, and adds many special short methods of calculation that increase efficiency, but the chief aim is to extend the child's power to make practical applications of arithmetic to life.

In the upper grammar grades, the child comes to feel at ease in performing the fundamental processes, only to find their applications more complex than ever. Hence the pressing responsibility of the grammar grades is to teach the children to think. They must be taught to understand the important social institutions with which arithmetical processes are associated, for, without this comprehension, they cannot reason out the successive steps to be taken in solving practical problems. Hence a large part of this text is devoted to the simple social and economic applications of mathematics.

Since the dominant problem in these higher grades is different from that of the lower years, it is necessary that the teacher modify the traditional spirit of mathematical teaching. The work should be formal only in the slightest degree. It would contribute greatly to effective instruction if the teacher would constantly remember that economic, as contrasted with formal, arithmetic deals chiefly with applications, and these should be kept vital to the children taught. While many arithmetical processes are very important to all pupils the whole country over, their applications are not. Each process has many varied uses. Every child may need to know every fundamental operation, but not every application of the same.

Recent investigations show that two urgent demands are being made by practically all school superintendents: (1) that fundamental processes be emphasized in the lower grades in order that early efficiency may result, and (2) that the social and economic applications of arithmetic be taught in the upper grades so that grammar school children will have an insight into the typical business practices of modern life. These texts are devised to meet both requirements.

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ARITHMETIC

EIGHTH YEAR

SECTION I

HOUSEHOLD PROBLEMS

Careful Spending

It has been said that while almost anybody can make money, it takes a wise man to keep it. This does not mean that a man is considered wise because he accumulates money at the expense of proper living; it means that more judgment is required to manage what is earned than to earn it.

Apportioning One's Income

The accompanying table gives the average per cents spent for rent, for food, and for clothing by certain city families consisting of two adults and two children, and having yearly incomes as specified:

Yearly Income	Expenditures for			
	Rent	Food	Clothing	Total
$600	19%	43%	13%	
$800	18%	42%	14%	
$1000	17%	40%	15%	
$1200	17%	37%	16%	

Written Problems

1. Copy the foregoing table, replacing the per cent of expenditure in each case by the amount in dollars. In the total column insert the sum of the three items.

2. Assuming that two children represent one adult, how many meals does a family of 2 adults and 2 children require in a year when three meals are eaten per day and fifteen meals are supplied to visitors during the year?

3. Find the average cost of food used at a meal by an adult, taking 3300 as the number of meals, when the income is (*a*) \$600, (*b*) \$800, (*c*) \$1000, (*d*) \$1200, and the sum expended is the per cent given in the preceding table. Give answers in cents and fractions of a cent.

Sight Problems

1. What is the price per pound of the lean meat in a rib roast costing 20 cents per pound, when 50 % of it is waste?

2. What is Mr. A's rent when he spends therefor 15 % of his annual income of \$1600?

3. What is the amount spent for clothing during the year by the family spending 16 % of an annual income of \$1400?

4. Find the yearly cost of the food for a family spending therefor 40 % of an annual income of \$900.

5. What is the cost of a quart of milk at the rate of $\frac{5}{8}$ ¢ per $\frac{1}{8}$ pt.?

6. At 3 ounces for a cent, what is the cost of a 1-pound loaf of bread?

7. When a medium-sized potato weighs 4 ounces, how many will there be in a bushel of 60 pounds?

8. Give the price per pound of each of the following at the given rates. Compare the result with the prices prevailing in the vicinity of the school.

a. $\frac{1}{2}$ ounce of coffee costs $1\frac{1}{2}$ ¢.
b. $\frac{1}{2}$ ounce of sugar costs $\frac{3}{16}$ ¢.
c. $\frac{1}{2}$ ounce of butter costs $1\frac{1}{2}$ ¢.
d. $\frac{1}{2}$ ounce of flour costs $\frac{1}{8}$ ¢.
e. $\frac{1}{4}$ ounce of cocoa costs 1 ¢.
f. $\frac{1}{2}$ ounce of tapioca costs 1 ¢.
g. 2 ounces of corn meal cost $\frac{3}{8}$ ¢.
h. 2 ounces of rice cost $1\frac{1}{4}$ ¢.
i. 2 ounces of canned beans cost 2 ¢.
j. 2 ounces of canned peas cost $2\frac{1}{4}$ ¢.
k. 2 ounces of hominy cost $\frac{3}{4}$ ¢.
l. 2 ounces of prunes cost 1 ¢.
m. 4 ounces of potatoes cost $\frac{1}{2}$ ¢.
n. 2 ounces of ham cost $3\frac{1}{2}$ ¢.
o. 4 ounces of roast beef cost $4\frac{1}{2}$ ¢.

9. When 6 prunes weigh 2 ounces, how many are there to the pound?

10. How many dates to the pound when each weighs $\frac{1}{4}$ oz.?

11. How many pounds do a dozen eggs weigh at 2 ounces each?

12. How many cups of coffee are made from a pound at the rate of $\frac{1}{2}$ oz. to the cup?

13. At the rate of $\frac{1}{3}$ oz. each, how many tablespoons of olive oil are there to the pound?

14. Give the cost of the berries in a dozen quart cans when $1\frac{1}{2}$ quarts of fresh berries at 10 ¢ per quart are required for one quart of canned berries.

Written Problems

1. Find the cost of a dozen codfish cakes containing 12 ounces of fish at 18 cents per pound, 3 eggs at 24 cents per dozen, and 12 potatoes weighing $\frac{1}{4}$ lb. each at 2 cents per pound.

2. At the rate of $1\frac{1}{2}$ ounces to 6 lettuce leaves, how many are there in a pound?

3. Find the cost of each of the following items in a quart jar of canned peaches, using:

a. A 16-qt. basket costing \$1.50 to make 13 jars.
b. $\frac{7}{8}$ lb. sugar to the jar @ 6¢ per pound.
c. A jar costing \$7 per gross of 144 jars.
d. 10 minutes of labor @ 12¢ per hour.

Give the results in cents correct to two decimal places.

4. How many hours are spent in a year in washing dishes (*a*) when 30 minutes are spent 3 times a day? (*b*) 40 minutes? (*c*) How many hours could be saved in a year by the use of conveniences that could save 15 minutes each time the dishes are washed?

5. (*a*) How many hours are spent in washing and ironing for 52 weeks at 12 hours per week? (*b*) What is the cost of the labor at the rate of \$7 for 60 hours? (*c*) Of the fuel and soap at 25 cents per week?

6. One hundred farmers contribute \$10 each to operate a laundry in connection with their creamery, using heat and power not required by the latter. A room is built at a cost of \$325 and machinery is installed at a cost of \$575. (*a*) How much is left for the purchase of soap and other supplies? (*b*) What is the cost of the labor for 52 weeks, an engineer being paid \$15 per week and two operators at \$7 each per week?

Checking Quantities and Extensions

Many persons who are careful in their selection of articles that they purchase, do not always get what they pay for. The seller may make a mistake in the weight, the quantity, or the cost. The careful housekeeper weighs or measures every article when it reaches her home. She verifies the correctness of each extension in a bill rendered, and also the footing. This does not necessarily imply doubt of the honesty of the seller; it means that she wishes a proper return of her money. She is just as prompt to call attention to an undercharge as to an overcharge.

High Cost of Living

When food prices are ascending, incomes increase at a smaller rate, and frequent adjustments between the various items of expenditure become necessary. By careful selection of food and skillful cooking, the good manager can continue to supply palatable and nutritious meals with the decreased amount she is able to allot to this item. She prolongs the life of the clothing without permitting it to become shabby. She sees that recreation is provided to the necessary extent, by directing it into less expensive channels, but without decreasing its amount or its benefit.

The School Kitchen

Following are typical meals cooked in a school kitchen, each containing the proper quantities and the right kinds of food for a man weighing 154 pounds, doing ordinary work.

The weight is that of the raw food ready for cooking. The cost of some materials will be less to the farmer's wife.

MONDAY'S BREAKFAST

Kind of Food	Weight	Cost
Stewed Fruit	2 oz.	2.25¢
Shredded Wheat	2	1.00
½ pt. Milk	2¼	0.62
Omelet, 1 egg	2	3.00
2 Rolls	2	2.00
1 cu. in. Butter	1	2.25
2 tb. sp. Coffee	½	1.12
1 tb. sp. Sugar	½	0.18
Totals		

TUESDAY'S BREAKFAST

Kind of Food	Weight	Cost
8 Steamed Dates	2 oz.	1.50¢
Cornmeal Mush	2	0.40
½ pt. Milk	2¼	0.62
2 slices Bacon	1	1.12
Creamed Potatoes	4⅞	1.52
3 slices Bread	3	1.00
1 cu. in. Butter	1	2.25
Coffee (2 tb. sp.)	½	1.12
Sugar (1 tb. sp.)	½	0.18
Totals		

MONDAY'S DINNER

Roast Lamb	4 oz.	4.75¢
½ pt. Rice	2	1.25
½ pt. Canned Peas	2	2.25
4 leaves Lettuce	1½	2.50
1 tb. sp. Oil	⅓	0.33
Tapioca and Apple Pudding	3	2.68
2 tb. sp. Coffee	½	1.12
1 tb. sp. Sugar	½	0.18
3 slices Bread	3	1.00
1 cu. in. Butter	1	2.25
Totals		

TUESDAY'S DINNER

Cream Soup (spinach)	5⅛ oz.	1.89¢
Macaroni and Cheese	5⅜	4.39
Stewed Corn	2	2.00
Stewed Tomatoes	2	2.00
Apple and Celery Salad	2½	3.00
Fruit Gelatine	1⅛	4.62
3 slices Bread	3	1.00
½ cu. in. Butter	½	1.12
Coffee (2 tb. sp.)	½	1.12
Sugar (1 tb. sp.)	½	0.18
Totals		

MONDAY'S SUPPER

Meat Roll	2 oz.	3.00¢
Potato Salad	4	0.50
Salad Dressing	1	0.75
3 slices Bread	3	1.00
1 cu. in. Butter	1	2.25
3 Canned Peaches	8	3.00
2 Sponge Cakes	2	2.00
Totals		

TUESDAY'S SUPPER

3 Codfish Cakes	5½ oz.	2.12¢
3 slices Bread	3	1.00
1 cu. in. Butter	1	2.25
Stewed Prunes	2	1.00
3 Cookies	1	1.00
Totals		

Written Problems

1. What is (*a*) the daily average weight of the food consumed? (*b*) The daily average cost? (*c*) Find the cost at this rate for the food of a family of 4 adults for a month of 30 days. Use prevailing prices.

2. The following shows the items of one type of a soldier's daily ration, with the cost of each to the government delivered at one of its posts.

(*a*–*q*) Find the cost of each item, giving the results in cents to four decimal places. (*r*) Find the cost of the entire ration. (*s*) Its total weight, taking the weight of a pint of vinegar as 1 lb., and that of a pint of sirup as 2 lb.

a. 16 oz. bacon at 24 ¢ per pound.
b. 18 oz. flour at $2\frac{3}{4}$ ¢ per pound.
c. .08 oz. baking powder at 9.7 ¢ per $\frac{1}{2}$-lb. can.
d. 2.4 oz. beans at 3.88 ¢ per pound.
e. 20 oz. potatoes at 1.47 ¢ per pound.
f. 1.28 oz. prunes at 5.94 ¢ per pound.
g. 1.12 oz. coffee at 14.75 ¢ per pound.
h. 3.2 oz. sugar at 4.165 ¢ per pound.
i. .5 oz. evaporated milk at 6.67 ¢ per 8-oz. can.
j. .02 pt. vinegar at 14.9 ¢ per gallon.
k. .64 oz. salt at .65 ¢ per pound.
l. .04 oz. pepper at 4.42 ¢ per $\frac{1}{4}$-lb. tin.
m. .014 oz. cinnamon at 15 ¢ per $\frac{1}{4}$-lb. tin.
n. .64 oz. lard at 12.98 ¢ per pound.
o. .5 oz. butter at 31.8 ¢ per pound.
p. .04 pt. sirup at 25 ¢ per gallon.
q. .014 oz. lemon extract at 30 ¢ per 8-oz. bottle.

3. Find the cost of rations for 1000 soldiers for 30 days.

4. Find in cents and a fraction the cost of each of the following types of dresses made in the sewing classes of a public school.

(a) Gingham Dress

4 yd. Gingham	@	\$.11¼
½ yd. Trimming	@	.12
½ doz. Buttons	@	.10
½ sp. Cotton	@	.05

(b) Princess Slip

2¾ yd. Cambric	@	\$.12½
4 yd. Trimming	@	.06
1¼ yd. Ribbon	@	.02½
½ doz. Buttons	@	.10
½ sp. Cotton	@	.05

(c) Dimity Dress

5 yd. Dimity	@	\$.12½
2¼ yd. Beading	@	.12
½ doz. Buttons	@	.10
¼ sp. Cotton	@	.05

(d) Graduation Dress

5 yd. Batiste	@	\$.15
4 yd. Lace	@	.12
½ doz. Buttons	@	.10
½ sp. Cotton	@	.05

5. How much can be saved (*a*) on clothes that cost \$240 by making them last half again as long through care in cleaning, patching, darning, etc.? (*b*) On farm implements that cost \$720 by making them last 2½ times as long by painting, repairing, oiling, sheltering from rain, etc.

6. Find the expenses shown by the following "Account with 14 Acres of Strawberries":

1 ton Fertilizer	\$ 34.00
12 M Plants @ \$ 2	
27 T. Manure @ \$ 2.20	
2⅗ T. Top Dressing @ \$ 50	
Carting & Spreading Fertilizer	17.82
Labor planting 216 days @ \$ 1.25	
Labor picking 360 days @ \$ 1.75	
Labor checking 30 days @ \$ 2	
Rent at \$ 10 per acre	

7. Find the returns from the foregoing field as follows:

3000 qt. Klondikes @ 6¢
6592 qt. Stevens @ 5¢
278 qt. Chesapeakes @ 8¢
32,528 qt. Gaudy Prize @ 7½¢

8. (*a*) What was the profit on the foregoing crop? (*b*) What was the average profit per acre?

9. Complete the following pay roll of a builder, in which double pay is given for work done outside of the regular hours. The check mark denotes 4 hours on Saturday and 8 hours on the other working days. The numbers above the check marks denote overtime. The wages specified are those paid for a day of 8 hours.

PAY ROLL FOR WEEK ENDING MAY 26, 1915

No.	Wages	Name	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Tot.	Hrs.	Rate	Amt.
1	$5.00	*W. Bauman*	✓	✓	✓	✓	✓	✓	44	44	62½¢	
2	4.50	*J. Bradley*	✓	$\overset{2}{✓}$	✓	✓	$\overset{3^2}{✓}$	✓	$\overset{5^2}{44}$	55	56¼	
3	4.50	*J. Byrnes*	6	✓	7	✓	$\overset{2}{✓}$	$\overset{4}{✓}$	$\overset{6}{41}$	53	56¼	
4	4.00	*E. Whitlock*	✓	5	6	$\overset{1}{✓}$	$\overset{1}{✓}$	$\overset{1}{✓}$	$\overset{3}{39}$		50	
5	4.00	*W. S. West*	✓	4	4	✓	✓	3			50	
6	4.00	*W. McGregor*	7	6	✓	7	✓	✓			50	
		Totals	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	—	(*i*)

Observe that J. Bradley's overtime of 5½ hours is doubled in combining it with the 44 regular hours.

10. Find the totals (*a*) to (*i*).

11. In the following page from the time book of the Tip-top Manufacturing Company a dash denotes absence, a check mark full time, 1¼ denotes ¼ day overtime.

PAY ROLL FOR WEEK ENDING OCT. 30, 1915

No.	NAME	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOT.	RATE	DUE
1	*A. Cooper*	✓	✓	✓	1½	✓	1½	6½	$28.50	$30.88
2	*B. Myers*	✓	1½	✓	✓	✓	1½	7	16.00	18.67
3	*C. Evans*	½	✓	✓	✓	✓	✓	5½	15.00	13.75
4	*D. Hofman*	✓	✓	1½	✓	—	✓	5½	12.50	
5	*E. Shaw*	½	—	✓	✓	✓	1½		10.00	
6	*F. Westervil*	—	1½	✓	✓	✓	✓		10.00	
7	*G. Gagnier*	✓	✓	½	✓	✓	✓		8.00	
8	*H. Laird*	1½	—	✓	1½	✓	✓		8.00	
9	*I. Seaton*	1½	✓	✓	✓	✓	1½		6.00	
10	*J. Bradley*	✓	✓	✓	✓	✓	—		6.00	
11	*K. Ward*	1½	✓	1½	✓	1½	✓		5.00	

Fill out the total weekly time of each employee, and the sum due at the given rate for 6 days' work.

12. Before going to the bank for the money, the cashier makes out a memorandum as follows, to ascertain the number of bills and of coins of each denomination needed for the pay envelopes of the several employees.

Complete the memorandum for the pay of the 11 employees and check the correctness of the totals.

No.	WAGES	$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1	$30.88	1	1				1	1	1		3
2	18.67		1	1	1	1	1		1	1	2
3	13.75		1		1	1	1	1			
4											
5											
etc.											
Total											

Keeping Bills and Receipts

When making a purchase the buyer should request a memorandum of the articles, prices, etc., whether they are paid for at the time or not. This is frequently made out on a narrow sheet of paper and is called a "sales slip." These memoranda enable the housewife that does not keep a daily account of her expenditures, to ascertain the totals for a week or a month.

The necessity for retaining receipts and receipted bills is obvious.

Household Expenses

The following table shows the living expenses of a family at three different periods:

Year	1912	1914	1916
Number in Family	2	3	4
Rent	$180	$180	$240
Food	200	400	480
Clothing	60	60	250
Fuel and Light	45	50	45
Furniture	—	10	12
Insurance	—	24	2
Health	—	36	24
Contributions	5	10	20
Recreation	5	8	20
Reading	12	8	10
Miscellaneous	63	60	85
Savings	?	?	?
Income	$600	$900	$1200

Household Accounts

A man's earnings do not always grow with his necessities or with the increasing price of articles required in his household. It is then that the intelligent housekeeper

puts forth additional efforts to make ends meet and perhaps to overlap a little. She is aided in this work by the keeping of accounts which show the character of her expenditures.

The following is one form of account in which twelve pages of a blank book are ruled to show the daily expenditures for each day of a month. On a thirteenth page, similarly ruled, are entered the totals for each month with a final footing giving the itemized summary of the year's receipts and expenditures.

JANUARY, 1916

	Receipts	Expenditures								Balance	Food			
		Rent, Repairs, Taxes, Fire Ins.	Food	Clothing	Life Ins., Dues	Recreation Health	Incidentals	Savings Bank	Totals		Butcher	Grocer	Etc.	Total
1														
2														
3														
4														
5														
6														
etc.														
31														
Total														
Decem.														
Increase														
Decrease														

In the first column of expenditures, the housekeeper living in her own house charges as rent the other items given, also interest on mortgage, etc. The column for food may be supplemented by other columns on the left giving the separate expenditures for meat, groceries, etc.

Written Exercises

1. The accounts of a housekeeper whose husband earned $15 per week and whose rent was $13 per month show that she paid $30 during the year for fuel, light, and ice; $.25 per month for newspapers; $2 per month on a sewing machine; $30 during the year for clothing; and $40 per month for food.

a. How much would remain for incidentals if she paid the doctor's bill of $50?

b. What per cent of the income was spent for rent?

c. For clothing?

d. How much more than 40 % of her income was spent for food?

2.

		EXPENSES	*Dr.*		*Cr.*	
1915						
Jan.	*1*	*Cash on hand*	*24*	*63*		
	2	*Rent for Jan.*			*13*	—
		Received	*13*	*50*		
	8	*Grocer*			*2*	*70*
		Baker				*70*
		Vegetables, Fruit				*95*
		Butcher			*2*	*10*
		Interest	*4*	—		
	9	*Shoes*			*3*	—
		Received	*13*	*50*		
		Balance			***33***	***18***
			55	*63*	*55*	*63*
	10	*Cash on hand*	*33*	*18*		
	11	*Savings bank*			*20*	—

3. Copy the foregoing account and extend it another week. Include the purchase of a ton of coal, the payment of the monthly gas bill, and other items, keeping the payments for the week under $25.

Written Problems

1. According to the Department of Labor, the average American working man, with a wife and family, spends his income as follows:

Food, 42.54 per cent; rent, 12.95; mortgage, principal and interest, 1.58; fuel, 4.19; lighting, 1.06; clothing, husband, 4.39; clothing, wife, 3.38; clothing, children, 6.26; taxes, .75; fire insurance, .20; life insurance, 2.53; dues in labor and other organizations, 1.17; religious purposes, .99; charity, .31; books and newspapers, 1.09; furniture and utensils, 3.42; amusements and vacations, 1.60; liquor, 1.62; tobacco, 1.42; sickness and death, 2.67.

a. What per cent remains for other purposes?

b. How many dollars are wasted yearly on liquors and tobacco by a man earning $75 per month?

c. Make a table showing the expenditures for each of the foregoing items by a person earning $1000 per year.

d. Make another table using the items shown in the one on page 11. Include under "rent" the mortgage payment and the one for taxes; combine the insurance payments, also those for clothing; etc.

2. Write from the book the "savings" in the table on page 11.

SECTION II

BUSINESS APPLICATIONS. PROPORTIONS AND ALIQUOT PARTS

Preparatory Exercises

Four persons engage a vehicle to take them 3 miles 4 miles, 5 miles, and 6 miles, respectively, the total expense of $7.20 to be apportioned among them according to the distance traveled by each.

Taking the sum of the distances, 18 miles, as the basis, the share of each might be determined by ascertaining the cost per mile and multiplying the result by the number of miles.

a. $\frac{\$7.20}{18} \times 3$ *b.* $\frac{\$7.20}{18} \times 4$ *c.* $\frac{\$7.20}{18} \times 5$ *d.* $\frac{\$7.20}{18} \times 6$

Another method is to multiply the total cost by the fraction of the total distance ridden by each.

a. $\$7.20 \times \frac{3}{18}$ *b.* $\$7.20 \times \frac{4}{18}$ *c.* $\$7.20 \times \frac{5}{18}$ *d.* $\$7.20 \times \frac{6}{18}$

Each of these fractions, $\frac{3}{18}$, $\frac{4}{18}$, $\frac{5}{18}$, and $\frac{6}{18}$, indicates the *ratio* between the distance traveled by an individual and the total distance.

In expressing each as a ratio, the colon (:) (which is one form of the sign of division) is used instead of the horizontal line employed in the fraction.

Thus, the ratio (*a*) of 3 to 18, (*b*) of 4 to 18, (*c*) of 5 to 18, and (*d*) of 6 to 18, is written

a. 3:18 *b.* 4:18 *c.* 5:18 *d.* 6:18

Sight Problems

1. If 3 oranges cost 9 cents, what will 8 oranges cost?

2. If 6 men require 11 days to do a piece of work, how many days will 22 men require to do the same work?

Proportion

Preparatory Exercises

Give the missing terms:

1. *a*. $\frac{3}{8}=\frac{9}{?}$ *b*. $\frac{3 \text{ oranges}}{8 \text{ oranges}}=\frac{9¢}{?¢}$ *c*. $\frac{3 \text{ lb.}}{9 \text{ oz.}}=\frac{48¢}{?¢}$

d. $\frac{22}{6}=\frac{11}{?}$ *e*. $\frac{22 \text{ men}}{6 \text{ men}}=\frac{\$66}{\$?}$ *f*. $\frac{1 \text{ bu.}}{3 \text{ pk.}}=\frac{?¢}{45¢}$

The equality of the ratios (*a*) 3 oranges and 8 oranges and (*b*) 9 cents and 24 cents may be written as a *proportion*, thus:

3 oranges : 8 oranges :: 9 cents : 24 cents,

or, omitting the denominations:

3 : 8 :: 9 : 24,

which is read "3 is to 8 as 9 is to 24."

It may also be written

3 : 8 = 9 : 24,

the double colon (::) being replaced by the sign of equality (=), which means the same, each signifying

$\frac{3}{8}=\frac{9}{24}$.

Observe that in the proportion

3 : 8 :: 9 : 24

the product of 3 and 24 (the *extremes*) is equal to the product of 8 and 9 (the *means*).

Finding a Missing Term

If 18 men can cut 40 acres of grass in a day, how many acres can be cut in a day by 15 men?

Since the ratio of the quantity cut should be equal to the ratio of the men employed, indicate the operations by the following proportion, in which x is used for the missing term:

$$\begin{matrix} 3 & & \\ \cancel{6} & 5 & 20 \\ \cancel{18}\text{ (men)} : \cancel{15}\text{ (men)} & :: & \cancel{40}\text{ (A.)} : x\text{ (A.)} \end{matrix}$$

Cancel 18 and 15, also 6 and 40. The product of the extremes, 3 and x, is equal to the product of the means, 5 and 20.

That is: $3x = 100$; which gives $x = 33\frac{1}{3}$ (A.) *Ans.*

Either extreme may be canceled with either mean.

Written Exercises

1. Find the missing term in the following proportion:

$$\begin{matrix} \text{Men} & \text{Men} & \text{Da.} & \text{Da.} \\ 13 & : 47 :: & 26 & : x \end{matrix}$$

Cancel 13 and 26.

2. Find the missing term:

$$3\text{ lb. } 6\text{ oz.} : 5\text{ lb. } 8\text{ oz.} :: \$x : \$4.40$$

Change the compound numbers to simple numbers, which gives the following proportion

$$\begin{matrix} \text{Oz.} & \text{Oz.} & \$¢ & ¢ \\ 54 & : 88 :: & x & : 440 \end{matrix}$$

Cancel 88 and 440. x equals 5 times 54. Insert the denomination in the result.

In forming a proportion the denominations may be placed above the numbers to indicate that abstract numbers are employed in the preliminary work, the denomination.

3. Find the value of x:

$$\begin{matrix} \text{Yd.} & \text{Yd.} & \$ & \$ \\ a.\ 18 & : 60 :: & 27 & : x \end{matrix} \qquad \begin{matrix} \text{Mi.} & \text{Mi.} & \text{Hr.} & \text{Hr.} \\ b.\ 40\frac{1}{2} & : 45 :: & x & : 40 \end{matrix}$$

Direct Proportion, Inverse Proportion

Preparatory Exercises

1. When 3 men can cut 7 acres of grass in a day, how many acres should 9 men cut at the same rate?

2. When 3 men require 8 days to mow a field, in how many days should 12 men mow it at the same rate?

In Ex. 1, the quantity cut is *directly* proportionate to the number of men. In Ex. 2, the time is *inversely* proportionate to the number of men.

In a builder's handbook it is stated that the load a beam can sustain varies *directly* according to the width of the beam, other things being equal, and *inversely* according to its length between its points of support.

This means that when a beam 2 inches wide will sustain 6 tons, one 4 inches wide will sustain 12 tons; and that when a beam 12 feet long will sustain 16 tons, one 16 feet long will sustain only 12 tons. A beam 6 inches deep will, however, sustain 4 times as much as one 3 inches deep, when both have the same length and width.

The solution of problems by means of proportion should be limited to such industrial and other types as are best treated in this way. Examples of this kind will be found in a later section.

The pupil must not assume, for instance, that the cost of insurance for 3 years is 3 times the cost for 1 year. He must remember that a boy who can walk 3 miles in 1 hour cannot walk 30 miles in 10 hours. He must not jump to the conclusion a pane of glass 9 times the size of a 5-cent one can be bought for 45 cents.

In the solution of the following problems employ any method you prefer.

Sight Problems

1. If 12 men can do a piece of work in 31 days, how long will it take 4 men to do the same work?

2. When a 54-acre field yields 186 tons of hay, (*a*) how many tons should an 18-acre field produce at the same rate? (*b*) If a 27-acre field yields 100 tons, how many tons more or less are obtained than from a corresponding area of the first field?

3. If 18 men can do a piece of work in 4 days working 10 hours a day, how many men will be required to do it in 10 days working 8 hours a day?

4. How many books at 75 ¢ each will be equal in cost to 93 books at $1.50 each?

5. A freight train traveling at the rate of 15 miles per hour requires 69 minutes between two stations. What time does an express train going 45 miles per hour require for the same trip?

6. If 2 lb. 3 oz. of butter cost 72 cents, what is the cost of 6 lb. 9 oz.?

7. If a train requires 3 hours to go 90 miles, (*a*) how many miles per hour should its speed be increased in order to make the trip in $\frac{2}{3}$ of the time? (*b*) By what fraction of 30 miles is the speed increased?

8. If 30 men can build a wall in 9 days, (*a*) how many fewer men would be required to do the work in 1 day more? (*b*) By what fraction is the time increased? (*c*) By what fraction is the number of men diminished?

9. If it costs $1664 to make a mile of road, what is the share of a man whose land extends 40 rods along the road?

Written Problems

1. When 14 men require 225 days to do a certain piece of work, how long would it take 50 men to do the same work?

SOLUTION BY PROPORTION

The time required to do work is inversely proportional to the number employed: twice as many men requiring one half the time.

50 men : 14 men : : 225 da. : x da.

SOLUTION BY ANALYSIS

If 14 men require 225 da.

1 man requires 225 da. $\times$ 14

50 men require $\frac{225 \text{ da.} \times 14}{50}$ Cancel

2. How long will a train require to go 97.6 miles at the rate of 3.2 miles in 8 minutes?

3. What should be the yield of $3\frac{1}{8}$ A. at the rate of 160 bushels to $8\frac{1}{8}$ A.?

4. If property worth \$15000 pays \$112.50 taxes, what should be the taxes on property worth \$12500?

5. If 9 pieces, each containing $27\frac{1}{2}$ yards, cost \$34.65, how many yards cost \$1.89?

6. By selling goods for \$70 I obtained 80% of their cost. At what price should they be sold to realize 120% of their cost?

7. (*a*) What is the cost of manufacturing an article retailed for \$1 if the retailer pays the manufacturer $\frac{10}{9}$ of the cost and charges the buyer $\frac{10}{9}$ of the price paid the manufacturer? (*b*) What improper fraction of the manufacturing cost does the consumer pay?

Aliquot Parts

A number is an aliquot part of another if it is an exact divisor of the latter. The aliquot parts of 100 are 50, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, $14\frac{2}{7}$, $12\frac{1}{2}$, etc.

Business men frequently employ aliquot parts in working some types of problems generally solved by analysis or by proportion.

8. If 33 acres of land produce 647 bushels of wheat, what should (*a*) 36 acres yield at the same rate? (*b*) 44 A.?

SOLUTION

(*a*) 33 acres yield 647 bushels
(Add) 3 acres yield $58\frac{9}{11}$ bushels ($\frac{1}{11}$ of 647 bushels)
36 $705\frac{9}{11}$ bushels *Ans.*

9. How many feet of ditch can 30 men dig in the same time that 33 men dig (*a*) 470 yards? (*b*) 560 yd.?

SOLUTION

(*a*) 33 men dig 470 yards
(Subtract) 3 men dig $42\frac{8}{11}$ yards ($\frac{1}{11}$ of 470 yards)
30 men dig $427\frac{3}{11}$ yards *Ans.*

10. If $\frac{3}{4}$ of a farm contains (*a*) 294 acres, how many acres are there in the farm? (*b*) 276 A.?

SOLUTION

(*a*) 3 fourths contain 294 acres
(Add) 1 fourth contains 98 acres ($\frac{1}{3}$ of 294 acres)
The farm contains 392 acres *Ans.*

Interest Tables

Clerks obliged to make numerous interest calculations use a book of tables. One style contains 360 pages, each giving the interest at various rates on specified amounts for the number of days stated at the top of the page. The accompanying table shows a portion of the page that gives the interest for 168 days.

168 Days

Principal	5%	6%	7%
1000	23.333	28.00	32.667
1100	25.667	30.80	35.933
1200	28.00	33.60	39.20
1300	30.333	36.40	42.467
1400	32.667	39.20	45.733
1500	35.00	42.00	49.00
1600	37.333	44.80	52.267
1700	39.667	47.60	55.533
1800	42.00	50.40	58.80
1900	44.333	53.20	62.067
2000	46.667	56.00	65.333

In using this portion of the table find the interest of \$250 for 168 da. at 5% by taking one fourth the interest of \$1000; the interest at 3% by taking one half the interest at 6%; the interest for 56 days by taking one third the interest for 168 days.

Sight Exercises

1. Give the principal that will yield in 168 days the interest yielded by

a. \$4000 in 42 da. *b.* \$8800 in 21 da. *c.* \$7200 in 28 da.
d. \$3000 in 84 da. *e.* \$4800 in 56 da. *f.* \$8400 in 24 da.

2. Using the table write only the answers to the following:

Give the interest on

a. \$4000 for 42 da., at 5% *b.* \$4800 for 56 da., at 6%
c. \$3000 for 84 da., at 6% *d.* \$7200 for 28 da., at 7%

Interest on Bank Loans

Written Exercises

1. Mr. Rorke borrows $475 from a bank Oct. 22, 1914. How much is required to pay the loan March 15, 1915, with interest at 5 %?

2. Find the interest on

a. $1230, at 4 %, from Jan. 15, 1914 to March 18, 1914
b. $2175, at 5 %, from Dec. 22, 1913 to April 14, 1914
c. $3260, at 6 %, from Aug. 17, 1915 to Jan. 10, 1916

For periods less than a year banks calculate interest for the exact number of days, generally considering the year to consist of 360 days in collecting interest and of 365 days in making interest payments.

A table in some such form as the following is used to ascertain the time between two dates that are not more than a year apart.

DATE TABLE

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	335	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	243	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	96	121	151	182	212	243	274	304	335	365

How to use the Table

To find the time from Sept. 3, 1914 to March 5, 1915, take the number at the intersection of the September line and the March column (181) which gives the number of days from Sept. 3 to March 3. To this add 2 days, the time from March 3 to March 5. *Ans.* 183 da.

The time from June 18, 1914 to Dec. 2, 1914 is found by taking the number at the intersection of the June line with the December column, 183 days (which is the time from June 18 to Dec. 18), and deducting 16 days (the time between Dec. 2 and Dec. 18), which gives 167 days as the result.

The time from Nov. 20, 1915 to May 3, 1916 is 181 da. − 17 da. + 1 da., this last being added for the extra day in February of a leap year. *Ans.* 165 da.

Sight Exercises

1. Give the number of days from
   - *a.* Jan. 16, 1914 to March 21, 1914
   - *b.* Dec. 20, 1913 to April 16, 1914
   - *c.* Aug. 10, 1915 to Jan. 15, 1916
   - *d.* April 20, 1914 to Dec. 12, 1914
   - *e.* Feb. 12, 1916 to Dec. 24, 1916
   - *f.* Sept. 15, 1915 to March 18, 1916

2. Give the interest on \$900 at 4 % to Jan. 4, 1916 from
   - *a.* Feb. 4, 1915
   - *b.* Aug. 4, 1915
   - *c.* March 1, 1915
   - *d.* Nov. 1, 1915

3. Give the amount of \$600 borrowed July 1, 1916, and paid with interest at 6 %.
   - *a.* Feb. 1, 1917
   - *b.* March 2, 1917
   - *c.* April 3, 1917
   - *d.* May 4, 1917

Written Exercises

1. Mr. Somers borrowed $3700 from the First National Bank Oct. 17, 1914. What sum will be required to repay it Jan. 9, 1915, with interest at 5 %?

PROCESS

The time is 84 days. From the table the interest for 168 days

on $2000 is	$46.667
on $1700 is	39.667
which gives the interest on $3700 for 168 da. as	$86.334

Interest for 84 da. is $\frac{1}{2}$ of that for 168 da.

2. Find the interest on

a. $2000 for 168 da., at 3 %
b. $3500 for 168 da., at 5 %
c. $1500 for 84 da., at 7 %
d. $2700 for 84 da., at 6 %
e. $1800 for 168 da., at $3\frac{1}{2}$ %
f. $3300 for 42 da., at 6 %
g. $1200 for 42 da., at 5 %
h. $2900 for 21 da., at $2\frac{1}{2}$ %
i. $1500 for 168 da., at $2\frac{1}{2}$ %
j. $2600 for 91 da., at 7 %

3. Find the amount of

a. $4000 from Jan. 2, 1915 to June 19, 1915 at 6 %
b. $4800 from Feb. 1, 1916 to Mch. 14, 1916 at 5 %
c. $3500 from Nov. 8, 1916 to Oct. 10, 1917 at $3\frac{1}{2}$ %
d. $2700 from May 7, 1915 to Dec. 3, 1915 at 3 %
e. $1200 from Apr. 8, 1916 to Mch. 10, 1917 at 6 %

Sight Review Problems

1. If a street is a mile long and 60 feet wide, how many miles must a watering cart travel to sprinkle it, if the cart sprinkles a strip 15 feet wide?

2. How many miles must be traveled by a steam roller 8 feet wide, in order to roll a road, 40 feet wide and a mile long, three times?

3. One boat's crew rows 30 strokes per minute and travels 24 feet each stroke, another rows 29 strokes per minute and travels 25 feet at each stroke. How much farther does one go in a minute than the other?

4. If a man obtains 5 sixths of the cost of an article when he sells it for $20, what must he charge for it to obtain 7 sixths of the cost?

5. At 99 cents each, how many baseballs can be bought for $24, and how much money will be left?

6. When 11 tenths of the cost of an article is $66, what is 12 tenths of its cost?

7. A train going 30 miles per hour requires 4 hours to make a trip. How many more miles per hour must it travel to make the trip in 1 hour less?

8. If 5 men dig a ditch 60 feet long in 10 days, how many days would it require 10 men to dig a ditch 30 feet long?

9. At $7\frac{1}{2}$ gallons to the cubic foot, find the capacity in gallons of a tank 5 feet long, 4 feet wide, 2 feet deep.

10. A can copy 15 pages in 3 hours and B can copy 15 pages in 5 hours; (*a*) how many pages can both together copy in 1 hour? (*b*) In what time can both together copy 40 pages?

Written Review Problems

1. A road is 45 feet wide. How many miles of it can be sprinkled in 10 hours by a water wagon traveling at the rate of $2\frac{1}{2}$ miles per hour, if the wagon sprinkles a strip 15 feet wide?

2. If a road is 48 feet wide, how long a strip can be rolled 3 times in 10 hours by a steam roller 8 feet wide moving at the rate of 2 miles per hour?

3. One boat's crew rows 33 strokes per minute and travels 29 feet per stroke; another rows 34 strokes per minute and travels 28 feet per stroke. If they are together at the start of a race, how far apart are they in 6 minutes?

4. If a man loses $16\frac{2}{3}\%$ when he sells an article for \$47.50, what price should he receive for it to gain $16\frac{2}{3}\%$?

5. Taking a meter as 39.37 inches, find the number of pieces 1 meter long that can be cut from 20 yards of tape, and the number of inches remaining.

6. If 10% is gained by selling furniture for \$171.60, what should be the selling price to make the profit 20%?

7. A steamer traveling 11 miles per hour takes $19\frac{1}{2}$ days to make a voyage. At what rate must it travel to make the voyage in 3 days fewer?

8. If 5 men can dig a ditch 60 feet long in 16 days, how many could dig 45 feet of the same ditch in 10 days?

9. At 231 cu. in. to the gallon, find the number of gallons that will fill a tank 4 ft. 1 in. long, 3 ft. 8 in. wide, and 2 ft. 6 in. deep.

10. A can copy $37\frac{1}{2}$ pages in $7\frac{1}{2}$ hours, and B can copy $13\frac{1}{2}$ pages in $4\frac{1}{2}$ hours; in what time can A and B together copy 280 pages?

To find the Principal, the Rate, or the Time

Sight Exercises

1. Mr. Jones receives $20 a year interest on money lent Mr. Brown at 5 %. How much did he lend?

2. Give the *principal* that will yield in 1 year at 5 %
   a. $20 *b.* $1.20 *c.* $24 *d.* $115 *e.* $.50

3. Give the *principal* that in 1 year will yield
   a. $20 at 4 % *b.* $1.20 at 3 % *c.* $4 at 5%

4. Give the *principal* that at 4 % will yield
   a. $20 in 2 yr. *b.* $16 in ½ yr. *c.* $48 in 180 da.

5. Mrs. Lynch receives $24 a year interest on $400 loaned Mrs. Burnet. What is the rate?

6. Give the *rate* at which $300 will yield in 1 year
   a. $18 *b.* $10.50 *c.* $15 *d.* $7.50 *e.* $9

7. Give the *rate* at which $100 will produce
   a. $12 in 2 yr. *b.* $10 in 2½ yr. *c.* $2.50 in 6 mo.

8. Give the *rate* at which, in 1 year,
   a. $200 yields $12 *b.* $300 yields $15

9. On a loan of $500 at 6 %, Mr. Melte receives $90 interest. For what time was the money lent?

10. Give the *time* required for $100 at 6 % to produce
    a. $12 *b.* $9 *c.* $1.50 *d.* $12.50 *e.* $27

11. Give the *time* in which, at 6 %,
    a. $200 yields $15 *b.* $100 yields $10

12. Give the *time* in which $300 will yield
    a. $12 at 6 % *b.* $18 at 4 % *c.* $27 at 4½ %

Compound Interest

A woman has $100 in a savings bank, drawing interest at 4% per year. (*a*) How much is there to her credit when a year's interest is added? (*b*) How much interest is due the second year on the first year's principal and interest?

By *compound interest* is meant the interest on the principal and on accrued interest.

Written Exercises

1. (*a*) Find the amount of $400 at 4% for 4 years, interest compounded annually. (*b*) Find the interest.

PROCESS

	Principal	$400.
The interest for each year is calculated on the amount at the end of the previous year.	Int. 1 yr.	16.
	Amount 1 yr.	$416.
	Int. 1 yr.	16.64
	Amount 2 yr.	$432.64
(*b*) The interest for 4 years is $467.94 − $400, or $67.94 *Ans.*	Int. 1 yr.	17.3056
	Amount 3 yr.	$449.9456
	Int. 1 yr.	17.9978
	Amount 4 yr. (*a*)	$467.94 *Ans.*

2. Find the interest on $300 at 5% for 4 years, compounded annually.

3. Mrs. Peck deposits $600 in a savings bank on Dec. 31, 1913. On June 30, 1914, 2% of this sum is added as a deposit. (*a*) How much is there to her credit in the bank at this date? (*b*) How much will there be to her credit Dec. 31, 1914, if the amount to her credit on June 30 is increased by an additional 2%?

PROCESS

Principal Dec. 31, 1913,	$600.	
Interest to June 30, 1914,	12.	(2% of $600)
Amount June 30, 1914,	$612.	
Interest to Dec. 31, 1914,	12.24	(2% of $612)
Amount Dec. 31, 1914,	$624.24	

4. Find the amount of each of the following at the end of the year, when interest is added half-yearly at the rate of 2%:

a. $200 *b.* $250 *c.* $300 *d.* $350 *e.* $550

Compound interest is practically never employed in ordinary business transactions. Even a savings bank that pays interest on the interest left as a deposit, rejects the cents in each new principal, although retaining it in the interest.

5. Mr. Harris deposited $600 in a savings bank on July 1, 1913. To how much interest is he entitled July 1, 1916, if interest at 2% is placed to his credit semi-annually?

PROCESS

Principal	$600.	
Int. ½ yr.	12.	
Amt. ½ yr.	$612.	(new principal)
Int. ½ yr.	12.24	
Amt. 1 yr.	$624.24	(new principal)
Int. ½ yr.	12.48	2% of $624
Amt. 1½ yr.	$636.72	(new principal)
Int. ½ yr.	12.72	2% of $636

In calculating the half-yearly interest, omit the cents, if any, in each new principal.

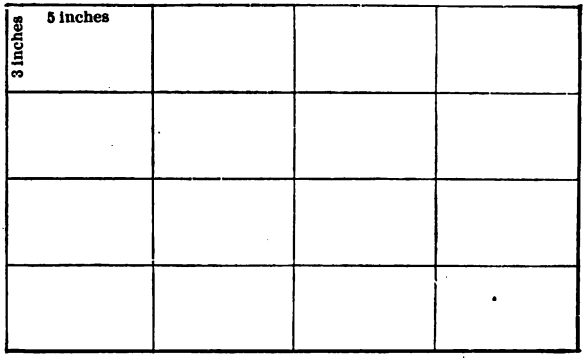
SECTION III

BUSINESS MEASUREMENTS

Area of a Rectangle

Preparatory Exercises

1. What is the length in inches of a row of four envelopes, each five inches long, placed end to end? What is the length in feet and inches?



2. What is the width in inches of four such rows, each envelope three inches wide, just touching each other? What is the width in feet?

3. How many envelopes are there? How many square inches are there in each envelope? How many square inches are covered by all of them?

4. How many envelopes 5 inches by 3 inches would cover the top of a table 4 ft. 2 in. by 2 ft. 6 in.?

Oral Problems

1. What is the difference between three square inches and three inches square?

2. What is the distance around a room that is 40 feet by 30 feet?

3. A garden is 12 feet long and 9 feet wide. How many bunches of flowers will it furnish, if it takes 3 square feet to furnish one bunch?

4. A room is 36 feet long and 30 feet wide. How many square yards in the floor?

5. How many yards is it around a room 15 feet long and 12 feet wide?

6. How many square inches in the surface of a sheet of paper 1 foot 8 inches long, 11 inches wide?

7. How many pieces of paper 2 inches square will exactly cover a slate 12 inches long, 8 inches wide?

Written Problems

1. How many boards 12 feet long, 6 inches wide will be required for a floor 8 yards long, 6 yards wide?

The floor is 24 feet long, 18 feet wide; its area in square feet is 18×24. The area of the board in square feet is $12 \times \frac{1}{2}$, or 6.

$$\text{Number of boards} = \frac{18 \times 24}{6}.$$

NOTE. — Save time and figures in examples involving multiplication and division by indicating the operations and using cancellation when possible.

2. How many bricks 8 inches by 4 inches will be needed for a walk 24 yards long, 6 feet wide, making no allowance for waste?

Area of top surface of one brick $= (8 \times 4)$ square inches. The length of the walk in inches $= 24 \times 3 \times 12$; width in inches $= 6 \times 12$. Area of walk in square inches $= 24 \times 3 \times 12 \times 6 \times 12$. Divide this by 8×4, the number of square inches in the top surface of a brick.

$$\text{Number of bricks} = \frac{24 \times 3 \times 12 \times 6 \times 12}{8 \times 4}.$$

NOTE. — Remember that the divisor and the dividend must be of the same denomination, square inches in this example.

3. How many paving tiles $\frac{1}{2}$ foot square will cover a hearth 6 feet long, 3 feet wide?

Make a diagram.

4. How many boards 12 feet long, 8 inches wide will be required for a close fence 120 yards long, 6 feet high?

5. Find the number of paving stones 9 inches by 3 inches, in a street 100 rods long, 10 yards wide.

6. Draw a rectangle 2 inches by 3 inches. Draw one twice the size. What are the dimensions of the latter? What are the dimensions of one four times the size?

7. A plot 100 feet by 100 feet is how many times as large as a plot 25 feet by 25?

8. A brick is 8 inches long, 4 inches wide, 2 inches thick. How many square inches are there in the surface of the widest face? In the surface of one side? In the surface of one end?

9. How many bricks laid on the widest face will be needed for a walk 288 inches long, 96 inches wide?

10. How many bricks laid on the side will be needed for a walk 24 feet long, 8 feet wide?

11. How many square feet are there in a roll of wall paper 24 feet long, 18 inches wide?

12. The owner of a piece of ground 200 feet wide, 300 feet long, divides it into lots 25 feet by 100 feet. How many lots are there?

13. Make table of square measure:

square inches (sq. in.)	= 1 square foot (sq. ft.)
square feet	= 1 square yard (sq. yd.)
square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
acres	= 1 square mile (sq. mi.)

14. There are 160 square rods in an acre. How many square yards are there in an acre?

15. Give the dimensions, in yards, of a field that will contain just an acre. Of one that will contain two acres.

16. *a.* At $80 per acre what is the value of a field 80 rods long, 70 rods wide?

b. What will it cost to fence the field at 20¢ per running yard?

17. A man has a lot 100 feet by 200 feet. How many square feet will he have left for a garden after he builds a house 25 feet by 60 feet?

18. One wall of a room is 24 feet long and 12 feet high. There is a door in it 8 feet high, $4\frac{1}{2}$ feet wide. How many square yards of plastering will be needed to cover the wall?

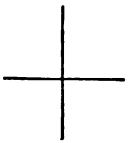
19. What would be the cost of painting 1800 feet of fence 6 feet high at 15 cents per square yard?

20. What is the length of a rectangular field 60 rods wide that contains 60 acres?

21. A farm is one mile square. How many 40-acre fields does it contain?

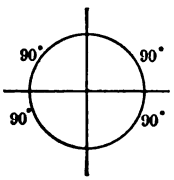
22. How many acres in a field in the shape of a triangle whose base and perpendicular measure 40 rods each?

Angles



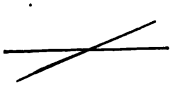
When two intersecting lines make four equal angles, the lines are said to be *perpendicular* to each other and each of the angles is called a *right angle.*

Angles are measured in degrees, the size of the angles depending on the portion of the circumference of a circle embraced between the lines forming the angle when the intersection of the lines is at the center of the circle.



A portion of the circumference of a circle is called an *arc.*

When two lines intersect obliquely, two of the angles are smaller than right angles and two of them are larger. An angle smaller than a right angle is called an *acute* angle; one larger than a right angle is called an *obtuse* angle.



Triangles

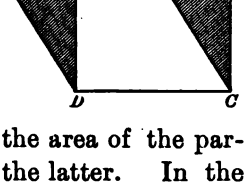
Considering the length of their sides, triangles are *equilateral*, having three equal sides; *isosceles*, having two equal sides; *scalene*, having unequal sides.

The three angles of an equilateral triangle are equal, each containing 60°. The angles opposite the equal sides of an isosceles triangle are equal, the side opposite the third angle being called the *base*, regardless of its position.

A triangle containing a right angle is called a *right triangle;* one containing an obtuse angle is called an *obtuse-angled* triangle; one containing three acute angles is called an *acute-angled* triangle.

Area of a Parallelogram

The parallelogram $ABCD$ may be changed to an equivalent rectangle by cutting along the line Dx, which is perpendicular to AB, and transferring the right triangle AxD to the place marked DyC, thus forming the rectangle $xyCD$.



The area of the rectangle $xyCD$ is the product of DC by xD, the base of the parallelogram by its altitude.

The lines whose lengths determine the area of the parallelogram are called *dimensions* of the latter. In the rectangular parallelogram $xyCD$, the dimensions are any two adjacent sides; xy and yC, yC and CD, etc. In the parallelogram $ABCD$, the base, AB or DC, is one dimension, and the perpendicular distance between them, xD or Cy, is the other.

The perpendicular distance between two sides of a parallelogram is called its *altitude*. A side that is perpendicular to the altitude is called the *base*. The dimensions of a parallelogram are the base and the altitude.

$$\textit{Area of parallelogram} = \textit{Base} \times \textit{Altitude}$$

Written Exercises

1. (*a*) Find the length of AD when Bx measures 8 rods and Ax measures 15 rods. (*b*) What is the area of the parallelogram when CD measures 24 rods?

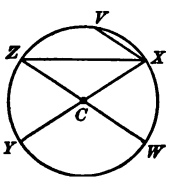
2. Find the altitude of a 14-acre field in the form of a parallelogram, the length of the base being 56 rods.

3. To a convenient scale draw a parallelogram in which Dx measures 8 rods, CD 24 rods, and Ax 6 rods. (*a*) How long is AB? (*b*) What is the area of the parallelogram?

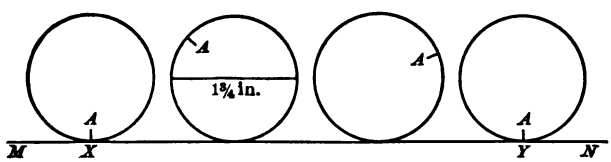
Circumference of Circle

A *circle* is a surface bounded by a curved line called the *circumference*, each point of the latter being at the same distance from the *center*.

The *diameter* of a circle is any straight line extending from one point in the circumference to another and passing through the center; a straight line between two points in the circumference but which does not pass through the center is called a *chord*. XY and ZW are diameters, XZ and XV are chords. The *radius* of a circle is a line extending from the center to the circumference, and is one half the diameter. CX, CW, CY, and CZ are radii.



The ratio between the diameter and the circumference of a circle may be experimentally determined as follows:



Make a cardboard circle having a diameter of $1\frac{3}{4}$ inches and mark a point A on its circumference. Roll it along a line beginning at X. When A again reaches the line, mark the point Y. The length of XY will be found to be $5\frac{1}{2}$ inches.

The ratio of $1\frac{3}{4}$ inches to $5\frac{1}{2}$ inches is 7 to 22, which is approximately the ratio of the diameter to the circumference.

Circumference of circle $= 3\frac{1}{7} \times$ *Diameter*

Sight Exercises

1. Give the circumference of a circle having a diameter of (*a*) $3\frac{1}{2}$ yd. (*b*) 7 ft. (*c*) 14 rods. (*d*) 21 meters.

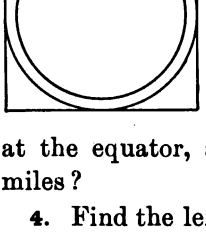
2. Give the circumference of a circle having a radius of (*a*) $3\frac{1}{2}$ mi. (*b*) 6 meters. (*c*) 7 rods. (*d*) 3 in.

3. Give the diameter of a circle having a circumference of (*a*) 22,000 mi. (*b*) 44 ft. (*c*) 220 yd. (*d*) 11 in.

Written Problems

1. The inner circumference of a circular track is 1 mile (1760 yards). What is the diameter in yards?

2. When the inner circumferenee of a circular track is $\frac{1}{2}$ mile, (*a*) what is its diameter in yards? (*b*) What is the outer diameter, if the track is 7 yards wide? (*c*) How much greater is the outer circumference than the inner one? (*d*) What is the length of one side of a square that will exactly contain the track?



3. What is the diameter of the earth at the equator, assuming the circumference to be 25,000 miles?

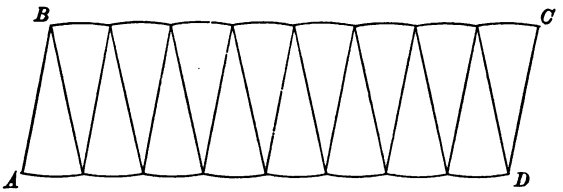
4. Find the length of a degree of longitude at the equator, there being 360 degrees in the circumference.

5. A bicycle wheel has a diameter of 2 ft. 4 in. (*a*) How many feet will the bicycle travel in each revolution of the wheel? (*b*) How many revolutions must the wheel make in going a mile?

6. A cog wheel 14 inches in diameter has 66 cogs. What is the distance between the cogs at the top, assuming the distance to be equal to the width of a cog?

Area of Circle

Cut a paper circle into as many equal parts as possible. Arrange the pieces as shown below to form a parallelogram as nearly as possible.



As the number of pieces increases, the nearer to a rectangle does the parallelogram become, so that when the number of pieces is indefinitely large, the figure is a rectangle.

The height of this rectangle is the radius of the circle; the length of the rectangle is one half the circumference of the circle, or $3\frac{1}{7}$ times the radius; the area is, therefore, $3\frac{1}{7}$ times the square of the radius.

Area of circle $= 3\frac{1}{7} \times$ *Square of radius*

By the square of a number is meant the product of the number by itself. The square of 8 is expressed 8^2, and is equal to 8×8, or 64.

Written Exercises

1. (*a*) Find the area of a circle whose radius is 42 feet. (*b*) Find the area of a circle whose diameter is 42 feet. (*c*) How do the areas compare?

2. The circumference of a circle is 220 yards. (*a*) Find the diameter. (*b*) The radius. (*c*) The area.

Capacity and Volume

Preparatory Exercises

1. How many 1-inch cubes would cover the bottom of a rectangular box 8 inches long and 6 inches wide, inside measurement?

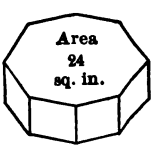
2. How many layers would be required to fill a box 4 inches high?

3. What is the capacity in cubic inches of a box $8'' \times 6'' \times 4''$?

The sign $''$ denotes inches, $'$ denotes feet, $\times$ denotes "by."
$8'' \times 6'' \times 4''$ is read 8 inches by 6 inches by 4 inches.

4. What is the volume of a rectangular block $8'' \times 6'' \times 4''$?

5. The bottom of an octagonal candy box has an area of 24 square inches. When it is 4 inches high, what is its capacity?



6. What is the volume of a marble block 4 inches high whose bottom and top are equal hexagons, each having an area of 21 square inches?

7. How many cubic inches of water will a tumbler hold when the inside area of its circular bottom is 10 square inches and it is 4 inches high, the opening at the top having the same area as at the bottom?

8. How many cubic feet of marble are there in a block whose bases are circles containing 20 square feet each, the distance between the bases being 10 feet?

9. (*a*) At $7\frac{1}{2}$ gallons to the cubic foot, what is the capacity of a cylindrical tank 10 feet deep and having a base containing 20 square feet? (*b*) Give the capacity in bushels of a bin of this size, taking $\frac{4}{5}$ bu. to the cu. ft.

Prisms and Cylinders

In mensuration, a box, a tumbler, a block, etc., is called a *solid*. The term *volume* means its capacity, its cubical contents, etc.

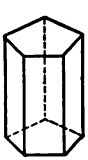
Each of the boxes and blocks in the first six of the foregoing examples is a *prism*. One of them is called an *octagonal* prism, another a *hexagonal* prism, and the others *rectangular* prisms.

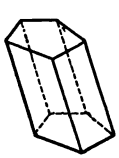
A rectangular prism has six *plane faces* and twelve *edges*. Two of its opposite faces are called its *bases*, which are parallel to each other and equal in area. Each edge is perpendicular to an adjacent one.

In the hexagonal prism, the two equal parallel faces are hexagons, and the remaining six, called the *lateral* faces, are rectangles; in the octagonal prism two are octagons and the eight lateral faces are rectangles. The hexagons in the former are called the bases, and the octagons in the latter.

The tumbler and the block in Examples 7 and 8 are called *cylinders*, the bases in each of which are equal circles. A cylinder has two plane faces and one curved face.

The foregoing are called *right* prisms or cylinders, each lateral edge in the former being perpendicular to the adjacent edge of the base.





The accompanying figures show a right pentagonal prism and an oblique one. Each of the ten lateral faces is a parallelogram, the five in the former being rectangles and the five in the latter being rhomboids.

Unless the term *oblique* is used, a right prism or cylinder is intended.

Volume of Prism or of Cylinder

Volume of right prism (or cylinder) = Area of base × Altitude.

Written Problems

1. Find (*a*) the area of the base and (*b*) the volume of a square prism 8 feet high, the sides of the base measuring 5 ft. 6 in. each.

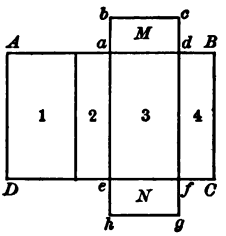
2. A cylindrical vessel is 7 inches in diameter and 6 inches high. (*a*) What is the area of its base? (*b*) What is its capacity? (*c*) How many gallons of water would it hold? (Take 231 cu. in. to the gallon.)

3. A cubical block of granite 7 feet each way was made into the largest cylinder possible. (*a*) How many cubic feet of granite were there in the original block? (*b*) How many cubic feet of granite are there in the cylinder? (*c*) What fraction of the area of the base of the cube is the area of the base of the cylinder? (*d*) What fraction of the block was cut away?

4. A piece of iron 6 inches square and 6 inches thick has a cylindrical hole bored through it $3\frac{1}{2}$ inches in diameter. (*a*) How many cubic inches of iron were there in the original piece? (*b*) How many cubic inches of iron are removed in making the hole? (*c*) How many cubic inches of iron remain?

5. A piece of timber 15 ft. long is 6 inches square at the ends. (*a*) Find its volume. (*b*) If its weight is one half that of an equal volume of water, how heavy is it? (A cubic foot of water weighs 1000 oz.) (*c*) Find the weight of a piece 32 ft. long and with ends 1 foot square.

Development (Pattern) of Prism



The illustration shows the *development* of a right prism. This may be cut from cardboard, the edges folded and fastened with gummed paper.

The four lateral faces together form a rectangle $ABCD$, the base of which, $DefC$, is the perimeter of the base of the prism and the altitude of which, AD, is the altitude of the prism.

By the *convex surface* of a solid is meant the sum of the lateral surfaces. The *entire surface* includes also the surface of the two bases.

Preliminary Exercises

1. When BC is 6 inches and Cf is 2 inches, what is the area of $dBCf$?

2. What is the area of rectangle No. 2?

3. What is the area of *adfe* when *ad* is 4 inches?

4. What is the area of rectangle No. 1?

5. What is the combined area of rectangles Nos. 1, 2, 3, and 4?

6. What is 6 times $(4 + 2 + 4 + 2)$?

7. What is the value of $(6 \times 4) + (6 \times 2) + (6 \times 4) + (6 \times 2)$?

8. What is the area of $ABCD$?

Convex surface of prism = Perimeter of base × Altitude.

Convex Surface

Sight Exercises

1. Give the perimeter of one base of each of the following prisms, when the bases are, respectively,

a. Equilateral triangles, each side 6 ft.
b. Parallelograms, each side 4 ft.
c. Rectangles, 2 ft. by 7 ft.
d. Right triangles; hypotenuse 5 ft., perpendicular 3 ft., base 4 ft.
e. Rhombuses (parallelograms), each side 5 ft.

2. Give the convex surface of each prism, when its altitude is 10 ft.

Entire Surface

The entire surface of a prism is the convex surface increased by the area of the bases.

Written Exercises

1. Find the entire surface of a square prism 7 in. high, the area of each base being 9 sq. in.

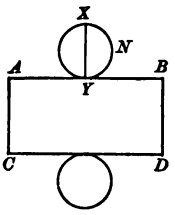
PROCESS

Area of base,	9 sq. in.	
Each side of base,	? in.	
Perimeter of base,	? in.	
Convex surface,	? sq. in.	Add.
Area of two bases,	? sq. in.	

2. How many square yards are there in the floor, the walls, and the ceiling of a room 16 ft. 6 in. long, 13 ft. 4 in. wide, and 9 ft. high?

Cylinder

The development of a right cylinder is a rectangle and two circles. The height of the former is the altitude of the cylinder, and its length is equal to the circumference of either base. The rectangle constitutes the convex surface.



Sight Exercises

Give the convex surface of each of the following cylinders:

a. Altitude, 7 in.; circumference of base, 21 in.
b. Altitude, 10 ft.; diameter of base, 7 ft.
c. Altitude, 5 ft.; radius of base, $3\frac{1}{2}$ ft.
d. Altitude, 9 in.; circumference of base, 9 in.
e. Altitude, $1\frac{1}{2}$ in.; diameter of base, 14 in.

Written Problems

1. A cylinder is 22 inches in circumference and 7 inches high. What is the entire surface?

PROCESS

Circumference of base,	22 in.	
Radius of base,	? in.	
Area of each base,	? sq. in.	
Of both bases,	? sq. in.	} Add
Convex surface,	? sq. in.	} Add

2. At 50¢ a square yard find the cost of polishing a granite shaft 10 ft. high, $3\frac{1}{2}$ ft. in diameter, omitting both ends. (The convex surface.)

Written Review Problems

1. Furniture catalogued at $60 is bought by a dealer at 40% discount. If he sells it at the catalogue price, what is his per cent of profit?

2. What per cent does A gain on an article that cost him (*a*) 30% below the price he obtains for it? (*b*) 50% below the price he obtains for it? (*c*) 25% below the price he obtains for it?

An article bought at 30 per cent below the selling price is bought at 70 per cent of the selling price.

The cost is, therefore, 70 per cent, and the selling price is 100 per cent. Ignoring the term "per cent," the cost is 70 and the selling price is 100. The profit is 30 on a cost of 70; etc.

3. An article costing $100 is marked 50% above cost. The dealer sells it at 50% below the marked price. (*a*) How many dollars does he lose? (*b*) What per cent does he lose?

4. If goods costing $200 are marked 20% above cost and are sold at 20% below the marked price, (*a*) what is the loss? (*b*) What per cent is lost?

5. If a farmer raised 450 bushels of wheat in 1909, 20% less in 1910, and 20% more in 1911 than he raised in 1910, (*a*) how many bushels did he raise in 1911? (*b*) What was the decrease in 1910 below 1909 in bushels? (*c*) What was the increase in 1911 over 1910 in bushels? (*d*) What was the per cent of increase or decrease of 1911 compared with 1909?

6. The following are the receipts of grain for a week:

Wheat 378 bu., corn 896 bu., barley 64 bu., rye 282 bu., oats 380 bu.

(*a*) Find the total receipts, and (*b*) the per cent of the total represented by each variety.

7. In a year a farmer sold \$4000 worth of produce, which cost him \$2400 in labor, etc. The next year his sales increased \$1000. What should be the increase in the expenses, in order that the rate of profit might be the same?

8. Find the cost of making a walk 2 rods long and 6 feet wide at 75 cents a square yard.

9. At the rate of 99 steps per minute how long would it take a regiment to march 6 miles, if each step measures $2\frac{1}{2}$ ft.?

10. To sell an article costing \$2.40 entails an expense of 5% of the cost. What is the selling price if the profit is $8\frac{1}{3}$% on the cost and the expense together?

11. A dealer buys eggs at 15 cents per dozen and sells them at 15 for 25 cents. To make a profit of \$6, (*a*) how many eggs must he sell? (*b*) How many cases of 30 dozen each?

12. A man has $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of his money left. (*a*) What fraction of his money is left? (*b*) If he has \$36 left, how much had he at first?

13. A person sells $\frac{1}{3}$ of his pigs, then $\frac{1}{5}$ of the remainder, then $\frac{1}{10}$ of the remainder. (*a*) What fraction of the original number is left? (*b*) If there are 72 pigs then left, what number had he originally?

14. A buyer is allowed discounts of $33\frac{1}{3}$, 20, and 10% on an article. Find (*a*) the equivalent single discount. (*b*) The equivalent per cent of the catalogue price. (*c*) The catalogue price, when the net price paid is \$96.

15. The table on the following page shows the results of experiments in raising oats on 5 plots of 1 acre each by 11 different farmers. No fertilizer was used on Plot 1; different ones were tried on each of the others.

TABLE

FARM	No FERTILIZER		*A* MIXTURE		*B* MIXTURE		*C* MIXTURE		*D* MIXTURE	
	Grain lb.	Straw lb.	Grain lb.	Straw lb.	Grain lb.	Straw lb.	Grain lb.	Straw lb.	Grain lb.	Straw lb.
A	1904	2912	2352	3696	1907	3024	2577	4144	2355	3920
B	2243	5040	2243	5600	1993	4480	2018	5376	2352	6596
C	1680	2128	1907	2464	2352	2464	2577	2912	2465	3136
D	1907	2016	1907	2128	2352	2576	2576	3024	2800	3024
E	1793	2912	1571	3024	1680	3248	2130	4256	2240	4704
F	1458	2912	2128	4256	2801	4480	3248	4704	3363	5040
G	1345	2464	1568	3024	1681	2688	1680	2688	1681	3136
H	2016	2016	2354	2464	2242	2240	2579	2912	2690	2912
I	1569	3472	2016	4256	1792	3920	2241	5040	2688	5600
J	2242	3808	2467	4704	2243	4368	2383	4928	2800	5600
K	2353	3472	2578	4266	2688	4256	2914	4928	3025	5264
Totals	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
Increase	——	——	(*k*)	(*l*)	(*m*)	(*n*)	(*o*)	(*p*)	(*q*)	(*r*)
Value of inc.	——	——	(*s*)	(*t*)	(*u*)	(*v*)	(*w*)	(*x*)	(*y*)	(*z*)
Total value of increase	——		I		II		III		IV	
Cost of fertilizer	——		$4.88		$4.80		$9.86		$12.58	
Profit	——		V		VI		VII		VIII	

Find the total weight of the grain and of the straw raised (*a*) to (*j*). Find the increase obtained by the use of the different mixtures (*k*) to (*r*). Find the value of the increase at the rate of 40 ¢ per bushel of 32 lb. for the grain (*s*), (*u*), (*w*), (*y*) and 50 ¢ per 100 lb. for the straw (*t*), (*v*), (*x*), (*z*). Find the total increase from each mixture, I to IV. Find the net profit following the employment of each by deducting its cost from the value of the increase in the crop, V to VII.

SECTION IV

EFFICIENCY IN CALCULATIONS

Business Methods

While each calling has its own particular set of "short-cuts" that are inapplicable in other lines, there are some general methods employed by all experienced business men to save unnecessary figures.

Written Exercises

1. Find the cost of 347 pounds of sugar at $5\frac{5}{8}$ cents per pound.

PROCESS

347		
$5\frac{5}{8}$		Find the product by $\frac{1}{8}$ by dividing the first product by 8. Do not write 8 as a divisor.
1735	Product by 5	
$216\frac{7}{8}$	Product by $\frac{5}{8}$	
$1951\frac{7}{8}$	*Ans.* $19.52	

2. Multiply:

a. $1247 \times 2\frac{2}{5}$ *b*. $2363 \times 5\frac{5}{6}$ *c*. $48 \times 10\frac{10}{11}$

3. Multiply $374\frac{2}{3}$ (*a*) by $\frac{3}{4}$. (*b*) by $\frac{5}{6}$.

PROCESS

(*a*)		(*b*)	
$374\frac{2}{3} \times \frac{3}{4}$		$374\frac{2}{3} \times \frac{5}{6}$	
$93\frac{2}{3}$	Deduct $\frac{1}{4}$	$62\frac{4}{9}$	Deduct $\frac{1}{6}$
281	*Ans.*	$312\frac{2}{9}$	*Ans.*

4. Find products:

a. $753 \times \frac{3}{4}$	*b.* $645\frac{1}{2} \times \frac{4}{5}$	*c.* $528\frac{1}{3} \times \frac{2}{3}$
d. $297 \times \frac{7}{8}$	*e.* $384\frac{1}{2} \times \frac{2}{3}$	*f.* $613\frac{1}{2} \times \frac{3}{4}$
g. $511 \times \frac{5}{6}$	*h.* $476\frac{1}{2} \times \frac{5}{6}$	*i.* $982\frac{2}{3} \times \frac{4}{5}$

5. Multiply 943 (*a*) by 24. (*b*) by $32\frac{1}{3}$. (*c*) by $15\frac{2}{3}$.

PROCESS

(*a*) 943×24	(*b*) $943 \times 32\frac{1}{3}$	(*c*) $943 \times 15\frac{2}{3}$
23575	$31433\frac{1}{3}$	$15716\frac{2}{3}$
22632 *Ans.*	$30490\frac{1}{3}$ *Ans.*	$14773\frac{2}{3}$ *Ans.*

Deduct 943 from (*a*) 25 times, (*b*) $33\frac{1}{3}$ times, and (*c*) $16\frac{2}{3}$ times itself.

6. Find products:

a. 357×49	*b.* $456 \times 32\frac{1}{3}$	*c.* $285 \times 15\frac{2}{3}$	*d.* $804 \times 49\frac{1}{2}$
e. 972×24	*f.* $456 \times 34\frac{1}{3}$	*g.* $285 \times 17\frac{2}{3}$	*h.* $561 \times 49\frac{3}{4}$
i. 972×26	*j.* $972 \times 24\frac{1}{2}$	*k.* $972 \times 24\frac{3}{4}$	*l.* $972 \times 24\frac{7}{8}$

7. Multiply $146\frac{3}{4}$ (*a*) by $1\frac{1}{8}$. (*b*) by $1\frac{1}{4}$. (*c*) by $1\frac{1}{6}$.

PROCESS

(*a*) $146\frac{3}{4} \times 1\frac{1}{8}$	(*b*) $146\frac{3}{4} \times 1\frac{1}{4}$	(*c*) $146\frac{3}{4} \times 1\frac{1}{6}$
$18\frac{11}{32}$	$36\frac{11}{16}$	$24\frac{11}{24}$
$165\frac{3}{32}$ *Ans.*	$183\frac{7}{16}$ *Ans.*	$171\frac{5}{24}$ *Ans.*

To $146\frac{3}{4}$ add (*a*) $\frac{1}{8}$, (*b*) $\frac{1}{4}$, (*c*) $\frac{1}{6}$ of itself.

Test (*a*) by multiplying $18\frac{11}{32}$ by 9, (*b*) by multiplying $36\frac{11}{16}$ by 5, (*c*) by multiplying $24\frac{11}{24}$ by 7.

8. Find products:

a. $144\frac{1}{2} \times 1\frac{1}{2}$	*b.* $216\frac{1}{2} \times 1\frac{1}{3}$	*c.* $324\frac{1}{2} \times 1\frac{1}{4}$	*d.* $486\frac{1}{2} \times 1\frac{1}{6}$
e. $376\frac{8}{9} \times 1\frac{1}{8}$	*f.* $252\frac{6}{7} \times 1\frac{1}{6}$	*g.* $108\frac{3}{4} \times 1\frac{1}{3}$	*h.* $276\frac{2}{5} \times 1\frac{1}{2}$

9. How many square feet are there in a steel plate 12 ft. 3 in. long, 4 ft. 4 in. wide?

PROCESS

$12\frac{1}{4}$ (ft.)
$4\frac{1}{3}$ (ft.)
49 4 times $12\frac{1}{4}$
$4\frac{1}{12}$ $\frac{1}{3}$ of $12\frac{1}{4}$
$53\frac{1}{12}$ (sq. ft.) *Ans.*

Express each dimension in feet and a fraction. Multiply $12\frac{1}{4}$ by 4, then by $\frac{1}{3}$, and combine the partial products. Test by changing the fractions to improper fractions, etc. $\frac{49}{4} \times \frac{13}{3}$.

10. Find products:

a. $12\frac{2}{3} \times 3\frac{1}{2}$ *b.* $126\frac{3}{4} \times 4\frac{1}{3}$ *c.* $245\frac{5}{6} \times 12\frac{1}{5}$
d. $12\frac{1}{4} \times 4\frac{1}{2}$ *e.* $132\frac{1}{2} \times 6\frac{1}{4}$ *f.* $376\frac{1}{8} \times 12\frac{1}{8}$

11. How many yards of silk can be bought for $\$374\frac{1}{2}$ when it costs (*a*) $\$\frac{3}{4}$ per yard? (*b*) $\$\frac{7}{8}$ per yard?

PROCESS

In (*a*) divide by $\frac{3}{4}$ by multiplying by $1\frac{1}{3}$; in (*b*) divide by $\frac{7}{8}$ by multiplying by $1\frac{1}{7}$.

(*a*) $\$374\frac{1}{2} \div \$\frac{3}{4}$
$124\frac{5}{6}$ Add $\frac{1}{3}$
Ans. $499\frac{1}{3}$ (yd.)

(*b*) $\$374\frac{1}{2} \div \$\frac{7}{8}$
$53\frac{1}{2}$ Add $\frac{1}{7}$
Ans. 428 (yd.)

Test (*a*) by multiplying $124\frac{5}{6}$ by 4, (*b*) by multiplying $53\frac{1}{2}$ by 8.

12. Divide $\$367\frac{1}{2}$ (*a*) by $66\frac{2}{3}¢$. (*b*) By $75¢$. (*c*) By $83\frac{1}{3}¢$. (*d*) By $87\frac{1}{2}¢$.

13. Find answers.

a. $846.6 \div .75$ *b.* $94.64 \div .875$
c. $846.6 \times .75$ *d.* $94.64 \times .875$

Sight Review Drills

1. Multiply

a. 48×25	*b.* 25×84	*c.* 124×25	*d.* 25×324
e. 49×25	*f.* 25×86	*g.* 168×25	*h.* 25×288
i. 32×25	*j.* 25×88	*k.* 165×25	*l.* 25×368

2. Give products:

a. $18 \times 33\frac{1}{3}$	*b.* $12\frac{1}{2} \times 88$	*c.* 16×125	*d.* 625×16
e. $24 \times 16\frac{2}{3}$	*f.* $37\frac{1}{2} \times 88$	*g.* $15 \times 333\frac{1}{3}$	*h.* 875×16
i. $36 \times 33\frac{1}{3}$	*j.* $62\frac{1}{2} \times 88$	*k.* $18 \times 166\frac{2}{3}$	*l.* 125×48

3. Multiply:

a. 96×99	*b.* 24×99	*c.* 96×49	*d.* 96×24
e. 99×99	*f.* 86×49	*g.* 84×24	*h.* 68×49
i. 88×99	*j.* 48×24	*k.* 85×99	*l.* 36×24

4. Give answers:

a. $99\frac{1}{2} \times 88$	*b.* $88 \times 49\frac{1}{2}$	*c.* $88 \times 24\frac{1}{2}$	*d.* $88 \times 12\frac{1}{2}$
e. $99\frac{3}{4} \times 88$	*f.* $88 \times 49\frac{3}{4}$	*g.* $88 \times 24\frac{3}{4}$	*h.* $88 \times 12\frac{1}{4}$
i. $99\frac{7}{8} \times 88$	*j.* $88 \times 49\frac{7}{8}$	*k.* $88 \times 24\frac{7}{8}$	*l.* $88 \times 12\frac{3}{8}$

5. Multiply:

a. $24 \times 99\frac{2}{3}$	*b.* $24 \times 49\frac{2}{3}$	*c.* $24 \times 24\frac{2}{3}$	*d.* $24 \times 16\frac{1}{3}$
e. $24 \times 99\frac{5}{6}$	*f.* $24 \times 49\frac{5}{6}$	*g.* $24 \times 24\frac{5}{6}$	*h.* $66 \times 16\frac{1}{3}$

6. Give products:

a. $48 \times 25\frac{1}{4}$	*b.* $48 \times 50\frac{1}{4}$	*c.* $36 \times 34\frac{1}{3}$	*d.* $48 \times 12\frac{3}{4}$
e. $48 \times 25\frac{1}{2}$	*f.* $48 \times 50\frac{1}{2}$	*g.* $36 \times 17\frac{2}{3}$	*h.* $48 \times 12\frac{5}{8}$

7. Multiply:

a. $32 \times 19\frac{1}{2}$	*b.* $16 \times 29\frac{1}{2}$	*c.* $24 \times 39\frac{1}{2}$	*d.* $24 \times 19\frac{2}{3}$
e. $32 \times 19\frac{3}{4}$	*f.* $16 \times 29\frac{3}{4}$	*g.* $24 \times 39\frac{3}{4}$	*h.* $24 \times 19\frac{5}{6}$

Special Products

Sight Exercises

To multiply two mixed numbers having the integer the same in each and the sum of the fractions 1, increase one of the integers by 1, multiply it by the other, and to the product annex the product of the fractions.

1. Multiply (*a*) $8\frac{1}{2}$ by $8\frac{1}{2}$. (*b*) $8\frac{1}{4}$ by $8\frac{3}{4}$.

In (*a*) the product $72\frac{1}{4}$ is 9 times $8 + \frac{1}{2}$ of $\frac{1}{2}$. In (*b*) the product $72\frac{3}{16}$ is 9 times $8 + \frac{1}{4}$ of $\frac{3}{4}$.

2. Give products:

a. $5\frac{1}{2} \times 5\frac{1}{2}$ *b.* $8\frac{1}{3} \times 8\frac{2}{3}$ *c.* $10\frac{1}{3} \times 10\frac{2}{3}$ *d.* $11\frac{1}{2} \times 11\frac{1}{2}$

e. $6\frac{1}{2} \times 6\frac{1}{2}$ *f.* $9\frac{1}{4} \times 9\frac{3}{4}$ *g.* $10\frac{1}{2} \times 10\frac{1}{2}$ *h.* $11\frac{1}{3} \times 11\frac{2}{3}$

3. Multiply (*a*) 8.5 by 8.5. (*b*) 7.3×7.7.

a. $8.5 \times 8.5 = (9 \times 8) + (.5 \times .5) = 72.25$

b. $7.3 \times 7.7 = (8 \times 7) + (.3 \times .7) = 56.21$

4. Multiply:

a. 2.5×2.5 *b.* 3.3×3.7 *c.* 4.4×4.6 *d.* 5.5×5.5

e. 2.8×2.2 *f.* 4.2×4.8 *g.* 6.3×6.7 *h.* 8.1×8.9

5. Multiply (*a*) 85×85. (*b*) 73×77.

In (*a*) annex to 72 (8×9) 25 (5×5), making the result 7225. In (*b*) annex to 56 (7×8) 21 (3×7), making the result 5621.

6. Give products:

a. 23×27 *b.* 34×36 *c.* 45×45 *d.* 56×54 *e.* 101×109

f. 55×55 *g.* 67×63 *h.* 88×82 *i.* 66×64 *j.* 112×118

Sight Review Problems

1. What is the area of a square having each side $6\frac{1}{2}$ inches long?

2. How many yards are there in 26 rolls of cloth each containing (*a*) 24 yd.? (*b*) 49 yd.?

3. A rectangle is 58 rods long and 52 rods wide; find its area.

4. A solid whose base is a 7-inch square is 41 inches high. What is its volume?

5. What is the cost of 96 bushels of wheat at $99\frac{7}{8}¢$ per bushel?

6. At 75¢ per yard, how many more than 123 yards can be bought for $123?

7. How many square inches are there in a rectangle $10\frac{7}{8}$ inches long, $10\frac{1}{8}$ inches wide?

8. If a man walks for 3 hours 45 minutes at the rate of $3\frac{1}{4}$ miles per hour, (*a*) how many miles has he walked? (*b*) How many miles and rods? (*c*) How many miles and rods would he walk if he walked for 3 hours and 30 minutes at the rate of 3 miles 160 rods per hour?

9. How many hours are there (*a*) in January? (*b*) In February 1915? (*c*) In February 1916?

10. Give the area of each of the following squares having sides

(*a*) 55 ft. (*b*) 65 yd. (*c*) 75 rd. (*d*) $8\frac{1}{2}$ mi. (*e*) $10\frac{1}{2}$ in.

11. Under the old tariff Mr. Martin paid 45% duty on an automobile valued at $4500. How much did he pay?

12. At the rate of 56 bushels of corn to the acre what is the yield of 54 acres?

13. (*a*) What fraction of a square foot is there in a pane of glass 8 in. by 6 in.? How many panes of this size are there in a box of glass containing 50 sq. ft.?

14. At $7\frac{1}{2}$ gallons to the cubic foot, how many cubic feet will 3000 gallons of oil occupy?

15. How many shares of stock at \$99 per share can be bought for (*a*) 93 hundred dollars, and how many dollars will remain? (*b*) For \$9306?

16. How many cubic yards are there (*a*) in 23 cu. ft.? (1 cu. yd. = 27 cu. ft.) (*b*) In 99 cu. ft.? (*c*) In $99\frac{3}{8}$ cu. ft.?

17. At $\$\frac{7}{8}$ per bushel, how many bushels of wheat can be bought (*a*) for \$1? (*b*) For \$147?

18. Find the cost of 248 bushels of wheat at $87\frac{1}{2}$ cents per bushel.

19. What per cent is gained on an article (*a*) bought for \$60 and sold for \$80? (*b*) Bought at 40% below the list price and sold at 20% below the list price? (*c*) Bought at 25% below the list price and sold at the list price?

20. What is the difference on a bill of \$100 between a discount of 40% and one of 20 and 20%?

21. In making out a check to pay for 75 shares of stock at \$99 per share, Mr. Byrnes found that his balance would be \$75 when the check was paid. How much had he in the bank?

22. When a dealer makes a profit of 25% by selling coffee at 30 cents a pound, what per cent would he make by selling it (*a*) for 27 cents per pound? (*b*) For 28 cents? (*c*) For 32 cents?

Written Review Problems

1. What is the cost of plowing an acre of land if one man with three horses plows $2\frac{1}{2}$ acres in 9 hours, at the rate of 20 cents per hour for the man's time and 10 cents per hour for that of each horse?

2. Find the cost when four horses are used and 4 acres are plowed in 9 hours.

3. By the use of a 40-horse-power engine a farmer plows 25 acres per day of 10 hours, at an expense of 15 cents per horse power for gasoline, 40 cents per hour for one man and 20 cents per hour for another, and $4 per day for the rent of the engine. Find the cost per acre.

4. Sheets 20 in. by 28 in. were used for a tin roof. (*a*) How many square inches are covered by each sheet if 1 inch of each side is turned in ("edged") when the sheets are soldered together? (*b*) How many square inches are used in the edging? (*c*) What per cent of each sheet is lost in the edging? (*d*) To cover each 468 square inches of roof, how many square inches of tin must be bought? (*e*) What per cent of 468 square inches must be bought?

5. (*a*) Find the dimensions of the piece of screen wire that will be required to make a screen for a window 36 inches wide 66 inches high when the wood frame is 3 inches wide and the wire overlaps the frame 1 inch on all sides. (*b*) How many square feet of screen wire will be needed?

6. (*a*) Find the dimensions of the piece of screen wire needed for one half a window, the space to be screened measuring 33 in. × 36 in., the wooden frame being 3 inches wide, and the wire overlapping 1 inch on the four sides? (*b*) How many square feet are required?

7. For the frame 2 pieces of wood 33 inches long are needed, and 2 pieces 36 inches long, each 3 inches wide. (*a*) How long a board 3 inches wide must be bought if the boards come only in even feet? (*b*) How many square feet are there in the surface of such a board? (*c*) What is the cost at $3\frac{1}{2}$ ¢ per square foot of surface? (*d*) Find the cost of the wood for 24 windows. (*e*) Of the screen wire for 24 windows at 3 cents per square foot.

8. (*a*) How many superficial square feet of wood are required for the frame of a screen door 7 ft. 6 in. high and 3 ft. 4 in. wide, using two strips for the length and three cross strips, one in the center, the strips being 4 inches wide? (*b*) Find the cost at $1\frac{1}{2}$ times $3\frac{1}{2}$ cents per superficial square foot, the wood being $1\frac{1}{2}$ times as thick as that used for the window frames.

9. (*a*) Find the dimensions of each of the two openings in the door screen which remain after the frame is made. (*b*) Find the dimensions of the screen wire to cover each when it overlaps 1 inch on each side. (*c*) Find the cost of the wire at 3 ¢ per sq. ft. (*d*) Find the cost of the wire for 4 doors. (*e*) Of the wood for 4 doors.

10. The valuation of a school district is $150,000. The school tax is 5 mills on the dollar, which maintains a school of 20 pupils. What is the cost per pupil?

11. Find the cost of a hen's feed for 180 days, consisting of .174 lb. grain per day at 1¢ per pound, .07 lb. ground bone at 1¢ per pound, and .02 lb. cabbage at $6 per ton.

12. (*a*) How many hens can be accommodated in a house 12 ft. wide by 32 ft. long, if each requires 6 sq. ft. of floor space? (*b*) How many sq. ft. of window are required at the rate of 1 sq. ft. to 16 sq. ft. of floor space?

Adding Mixed Numbers

Written Exercise

1. Find the sum of $18\frac{3}{4}$, $27\frac{5}{6}$, and $9\frac{1}{4}$.

PROCESS

Since $\frac{3}{4} + \frac{1}{4} = 1$, do not change these fractions to others having a common denominator.

Write the answer.

$18\frac{3}{4}$
$27\frac{5}{6}$
$9\frac{1}{4}$

Ans.

2. Write the results:

a.	*b.*	*c.*	*d.*	*e.*
$16\frac{3}{8}$	$56\frac{3}{7}$	$72\frac{7}{9}$	$123\frac{3}{5}$	$275\frac{5}{6}$
$48\frac{2}{3}$	$8\frac{4}{7}$	$63\frac{2}{9}$	$84\frac{7}{12}$	$63\frac{3}{4}$
$9\frac{5}{8}$	$35\frac{3}{8}$	$8\frac{3}{5}$	$17\frac{5}{12}$	$9\frac{1}{6}$

3. Add $4\frac{5}{7}$, $16\frac{3}{8}$, $8\frac{2}{5}$, and $37\frac{5}{8}$.

PROCESS

Since $\frac{3}{8} + \frac{5}{8} = 1$, omit them in getting the common denominator. Write $+1$ as a reminder that $1\frac{4}{35}$ (the sum of the fractions) must be increased by 1. Carry 2, therefore, to the whole numbers.

		35
	$4\frac{5}{7}$	25
	$16\frac{3}{8}$	×
	$8\frac{2}{5}$	14
	$37\frac{5}{8}$	× + 1
Ans.	$67\frac{4}{35}$	$\frac{39}{35} = 1\frac{4}{35}$

4. Find sums:

a. $63\frac{1}{2} + 8\frac{7}{8} + 35\frac{5}{9} + 5\frac{1}{2} + 23\frac{1}{8} + 16\frac{2}{5}$

b. $28\frac{2}{3} + 9\frac{2}{9} + 61\frac{5}{12} + 4\frac{1}{3} + 37\frac{3}{8} + 22\frac{7}{9}$

c. $15\frac{1}{6} + 7\frac{2}{5} + 81\frac{5}{6} + 1\frac{5}{8} + 56\frac{2}{3} + 18\frac{3}{5}$

d. $30\frac{3}{7} + 4\frac{4}{9} + 23\frac{2}{5} + 3\frac{4}{7} + 13\frac{7}{12} + 25\frac{3}{5}$

e. $50\frac{1}{4} + 6\frac{3}{8} + 75\frac{8}{10} + 7\frac{3}{4} + 42\frac{7}{10} + 14\frac{2}{9}$

5. Add $24\frac{2}{3}$, $47\frac{3}{4}$, $8\frac{1}{6}$, and $15\frac{2}{5}$.

PROCESS

Determine the least common denominator by inspection, when possible. Ignoring 3, which is contained in 6, think 12 as the least common multiple of 4 and 6. As 12 and 5 are prime to each other, 60 (their product) is the least common denominator.

	60
$24\frac{2}{3}$	
$47\frac{3}{4}$	
$8\frac{1}{6}$	
$15\frac{2}{5}$	

6. Find sums:

a. $128\frac{2}{3} + 74\frac{3}{4} + 80\frac{5}{6} + 7\frac{3}{8} + 125\frac{5}{9}$

b. $216\frac{1}{2} + 68\frac{4}{5} + 90\frac{2}{3} + 5\frac{4}{9} + 213\frac{1}{6}$

c. $342\frac{1}{4} + 35\frac{3}{7} + 50\frac{1}{2} + 9\frac{5}{6} + 162\frac{2}{3}$

7. Add $163\frac{7}{10}$, $29\frac{8}{15}$, $7\frac{16}{33}$, and $18\frac{17}{30}$.

PROCESS

When the least common denominator cannot be determined by inspection, write each denominator that is not a factor of any other.

$3)30 - 33$

$10 - 11$

$3 \times 10 \times 11 = 330$

In this case omit 10 and 15, since they are factors of 30.

		330
$163\frac{7}{10}$		231
$29\frac{8}{15}$	22	176
$7\frac{16}{33}$		
$18\frac{17}{30}$		187
	Ans.	

Divide 30 and 33 (the remaining ones) by 3, which is a factor of each. Since 10 and 11, the quotients, are prime to each other, the least common denominator is the continued product of the divisor and the quotients.

In changing the given fractions to 330ths, write the quotient of the latter by a denominator in the first column when necessary, and then write in the second column its product by the numerator.

8. Find sums:

a. $26\frac{3}{4} + 9\frac{4}{7} + 8\frac{5}{21} + 10\frac{8}{15}$ *b.* $1\frac{3}{8} + 3\frac{11}{12} + 5\frac{7}{18} + 9\frac{13}{24}$

Subtracting Mixed Numbers

Written Exercises

1. From $135\frac{8}{18}$ take (*a*) $89\frac{11}{12}$; (*b*) $77\frac{9}{10}$.

PROCESS

(*a*) $135\frac{8}{18} = 135\frac{26}{36}$, $89\frac{11}{12} = 89\frac{33}{36}$, *Ans.* $45\frac{23}{36}$. (*a*) Since $\frac{33}{36}$ is greater than $\frac{26}{36}$, increase the latter by 1, or $\frac{36}{36}$, writing 62, the sum of 26 and 36, alongside.

(*b*) $135\frac{18}{18} = 135\frac{65}{90}$, $77\frac{9}{10} = 77\frac{81}{90}$, *Ans.* $57\frac{74}{90} = 57\frac{37}{45}$. 155

In (*a*) an accountant would deduct $\frac{33}{36}$ from $1\frac{26}{36}$ by taking $\frac{33}{36}$ from 1 and adding the remainder, $\frac{3}{36}$, to $\frac{26}{36}$. In (*b*) he would take 81 from 90 and add 65 to the remainder.

$$36 + 26 - 33 = 36 - 33 + 26$$
$$90 + 65 - 81 = 90 - 81 + 65$$

2. Find remainders:

a. $236\frac{1}{4} - 112\frac{5}{6}$	*b.* $375\frac{2}{3} - 246\frac{5}{8}$	*c.* $821\frac{3}{5} - 179\frac{2}{3}$	*d.* $672\frac{4}{9} - 317\frac{7}{12}$
e. $300\frac{2}{5} - 106\frac{9}{10}$	*f.* $314\frac{5}{8} - 175\frac{7}{9}$	*g.* $470\frac{1}{6} - 206\frac{3}{5}$	*h.* $239\frac{7}{12} - 189\frac{5}{8}$

Decimals — Multiplication

To multiply a decimal by 10, 100, 1000, move the decimal point one, two, three places.

Sight Exercises

Give products:

a. 1.4×10	*b.* $.12 \times 100$	*c.* $.011 \times 1000$	*d.* 1.3×300
e. 1.4×20	*f.* $.12 \times 200$	*g.* $.011 \times 2000$	*h.* 2.2×400
i. 1.4×30	*j.* $.12 \times 300$	*k.* $.011 \times 3000$	*l.* 1.1×500
m. 1.2×40	*n.* $.12 \times 400$	*o.* $.022 \times 4000$	*p.* 3.2×200
q. 1.1×50	*r.* $.11 \times 500$	*s.* $.011 \times 5000$	*t.* 4.5×100

Written Exercises

1. Multiply 84.635 (*a*) by 20. (*b*) By 400.

PROCESS

(*a*) 846.35 × 2 = 1692.7$\emptyset$ *Ans.* In (*a*) multiply 84.635 by 10 by moving the decimal point one place to the the right, and in (*b*) multiply it by 100 by moving the decimal point two places to the right. Multiply the changed multiplicands by 2 and by 4, respectively. (*b*) 8463.5 × 4 = 33854.$\emptyset$ *Ans.*

2. Write products from the book:

a. 30 × 28.74	*b*. 600 × 1.487	*c*. 7000 × .2468
d. 40 × 3.465	*e*. 700 × 24.63	*f*. 8000 × 1.357
g. 60 × .1248	*h*. 800 × .1845	*i*. 9000 × 23.45

3. Multiply 87.4 (*a*) by 2.5. (*b*) By 12.5.

PROCESS

Some accountants prefer to change 2.5 and 12.5 to .25 and .125, respectively.

(*a*) 874 × .25 In (*a*) move the decimal point one place to the right in the multiplicand and one place to the left in the multiplier. In (*b*) move it two places to the right in the former and two places to the left in the latter. In (*a*) divide by 4; in (*b*) by 8. (*b*) 8740 × .125

4. Find answers:

a. 310.48 × 2.5	*b*. 620.96 × 1.25	*c*. 36 ÷ 1.25
d. 412.96 × .25	*e*. 715.76 × 12.5	*f*. 42 ÷ 12.5
g. 514.32 × 25	*h*. 844.32 × 125	*i*. 84 ÷ 12.5

5. Multiply:

Change multipliers to .75 or .875, then deduct $\frac{1}{4}$ or $\frac{1}{8}$.

a. 10.48 × 7.5	*b*. 12.96 × 8.75	*c*. 14.32 × 87.5
d. 12.96 × 7.5	*e*. 14.32 × 8.75	*f*. 10.48 × 87.5

Adding and Subtracting

Write only the answers:

1. Find the balances in the following cash accounts:

a.	*b.*	*c.*	*d.*
\$374.80	\$1234.56	\$ 817.74	\$8.47
97.25	683.59	3256.75	8.47
583.84	88.47	392.48	8.47
1247.56	205.68	176.69	8.47
185.13	1542.80	2216.45	8.47
?	?	?	?
\$3247.58	\$4000.00	\$7123.18	\$50.00

Multiplying and Subtracting

2. How much less than \$50 will be the cost of 5 coats at \$8.47 each?

PROCESS

\$50 − 5 times \$8.47 = \$7.65. Think 35 (5 × 7) and **5** (writing 5) are 40. Think 20 (5 × 4), 24 (carrying 4) and **6** (writing 6) are 30. Think 40 (5 × 8), 43 (carrying 3) and **7** (writing 7) are 50.

NOTE. Use the word "and" only in connection with the remainder that is to be written.

3. Write answers:

a. 896 − (9 × 97) *b.* 650 − (7 × 89) *c.* 737 − (8 × 85)
d. 528 − (6 × 83) *e.* 400 − (5 × 69) *f.* 617 − (6 × 98)

4. Change $\frac{5000}{847}$ to a mixed number.

PROCESS

$\frac{5000}{847} = 5\frac{???}{847}$ Write 5 as the whole number and 847 as the denominator of the fraction. Think 35(5 × 7) and **5** (writing 5) are 40; etc.

5. Write as mixed numbers:

a. $\frac{896}{97}$	*b.* $\frac{650}{89}$	*c.* $\frac{737}{85}$	*d.* $\frac{528}{83}$
e. $\frac{245}{82}$	*f.* $\frac{293}{37}$	*g.* $\frac{175}{23}$	*h.* $\frac{791}{92}$
i. $\frac{202}{35}$	*j.* $\frac{252}{37}$	*k.* $\frac{641}{84}$	*l.* $\frac{145}{19}$
m. $\frac{134}{15}$	*n.* $\frac{196}{25}$	*o.* $\frac{147}{22}$	*p.* $\frac{148}{29}$

Multiplying and Adding

6. What is the cost of 5 coats at \$8.47 each and a shawl at \$7.65?

PROCESS

(\$8.47 × 5) + \$7.65 = \$50.00 *Ans.*

Think 35(5×7), 40 (adding in 5); write 0. Think 20 (5×4), 24 (carrying 4), 30 (adding in 6); write 0. Think 40 (5 × 8), 43 (carrying 3), 50 (adding in 9); write 50.

7. Write answers:

a. 23 + (8 × 28)	*b.* (9 × 56) + 27	*c.* 53 + (7 × 84)
d. (6 × 57) + 35	*e.* 69 + (5 × 74)	*f.* (4 × 84) + 37
g. 27 + (5 × 35)	*h.* (6 × 37) + 30	*i.* 19 + (7 × 12)

8. Change $5\frac{765}{847}$ to an improper fraction.

PROCESS

$5\frac{765}{847} = \frac{????}{847}$ Write 847 as the denominator. Find the numerator by thinking 35 (5×7), 40 (adding in 5); write 0; etc.

9. Write as improper fractions:

a. $8\frac{23}{28}$	*b.* $9\frac{27}{56}$	*c.* $7\frac{53}{84}$	*d.* $6\frac{35}{57}$	*e.* $5\frac{69}{74}$
f. $8\frac{37}{41}$	*g.* $6\frac{18}{37}$	*h.* $9\frac{15}{23}$	*i.* $7\frac{18}{39}$	*j.* $5\frac{16}{47}$
k. $5\frac{27}{35}$	*l.* $6\frac{30}{37}$	*m.* $7\frac{12}{19}$	*n.* $8\frac{31}{32}$	*o.* $9\frac{15}{16}$

Abbreviated Division

10. A farmer raised 5280 bushels of corn on 83 acres. Find the average yield to the acre.

PROCESS

Ans. $63\frac{51}{83}$ bu.

$83\overline{)5280}$ bu.

300

51

Do not write the partial products.

Think 18 (6 × 3) and **0** (writing 0) are 18. Think 48 (6 × 8) 49 (carrying 1) and **3** (writing 3) are 52. To 30 (the first remainder) add 0, making 300 the second partial dividend. Proceed as before.

11. Divide without writing the partial products:

a. 8960 ÷ 97 *b.* 2931 ÷ 37 *c.* 4500 ÷ 39

d. 6507 ÷ 89 *e.* 1756 ÷ 23 *f.* 4718 ÷ 53

Lowest Terms

12. A tract containing 232 acres was sold for $2871. What was the price per acre?

PROCESS

$\$\frac{2871}{232} = \$12\frac{87}{232}$. To ascertain the possibility of expressing the fraction in lower terms, divide 232 by 87. Omit the quotient figure, 2, but subtract 2 × 87 from 232, which gives 58 as the remainder. Divide 87 by 58, which gives 29 as the remainder. Divide 58 by 29. Since this leaves no remainder, 29 is the greatest common divisor. Divide both terms of the fraction by 29.

87	232
29	58

13. Divide. Find answers in mixed numbers with fractions expressed in lowest terms. Use as few figures as possible.

a. 899 ÷ 203 *b.* 1221 ÷ 187 *c.* 2035 ÷ 176

d. 549 ÷ 204 *e.* 1961 ÷ 444 *f.* 2329 ÷ 221

Graphs

Numbers, which are usually represented by figures and by words, are sometimes represented by *lines*, especially to enable an interested person to compare at a glance relative quantities, or to determine the nature of numerical changes.

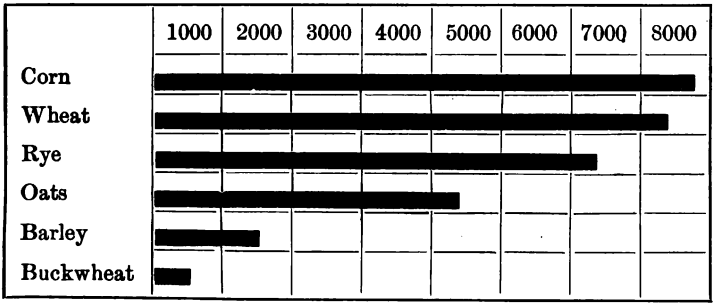
They are used to compare the population of different countries, their exports and imports, temperature, rainfall, etc. A general name for an illustration of this kind is a *graph.*

The accompanying graph shows the quantity of various kinds of grain received by rail during a month, as follows:

Corn 7810 bu.	Rye 6230 bu.	Barley 1475 bu.
Wheat 7250 bu.	Oats 4315 bu.	Buckwheat 420 bu.

each space representing 1000 bushels. Lighter lines are sometimes inserted to show each hundred.

RECEIPTS OF GRAIN DURING MAY, 1914



1. Make a similar graph to show the comparative number of pupils in the following grades:

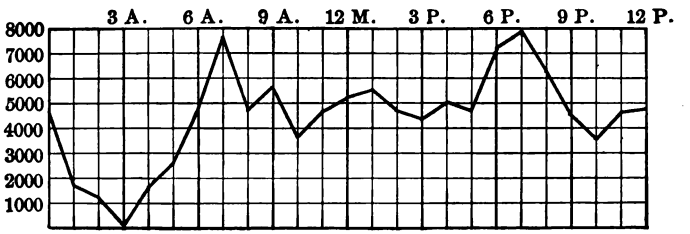
1st	345	4th	200	7th	110
2d	290	5th	170	8th	100
3d	250	6th	150		

Cross-ruled Paper

To save the time required to rule the necessary lines, cross-ruled paper is much used in making diagrams, graphs, etc. The paper is divided into squares of equal size by faint lines $\frac{1}{4}$ in. apart, $\frac{1}{8}$ in., etc. Some forms employ a darker line to denote every tenth one, as an additional help.

The use of this "squared" paper is shown in the following graph, which indicates the changes in the number of passengers carried by a trolley line during each hour of the twenty-four.

PASSENGERS CARRIED, MAY 25, 1916



The number of passengers indicated by the foregoing diagram was as follows for each hour:

A.M.	12–1, 1550;	4–5, 2750;	8–9, 5600
	1–2, 1230;	5–6, 4800;	9–10, 3790
	2–3, 475;	6–7, 7925;	10–11, 4900
	3–4, 1880;	7–8, 4710;	11–12, 5300
P.M.	12–1, 5600;	4–5, 4900;	8–9, 4550
	1–2, 4810;	5–6, 7200;	9–10, 3800
	2–3, 4190;	6–7, 7900;	10–11, 4890
	3–4, 5005;	7–8, 6220;	11–12, 4920

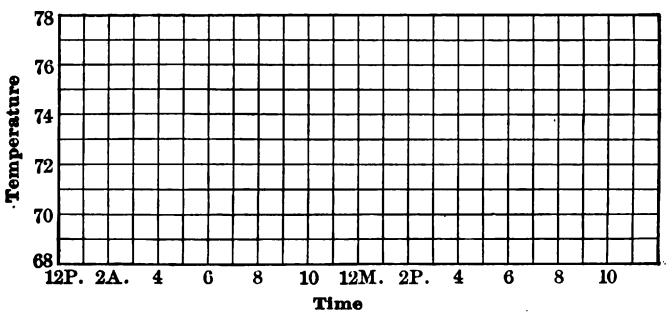
Written Exercises

1. Find the number carried (*a*) before noon. (*b*) From noon to midnight. (*c*) All day.

2. On squared paper make a graph to show the changes in temperature indicated by the following record for July 28, 1915.

12 P.M. — 71	6 A.M. — 71	12 M. — 75	6 P.M. — 74
1 A.M. — 70	7 A.M. — 71	1 P.M. — 76	7 P.M. — 73
2 A.M. — 68	8 A.M. — 72	2 P.M. — 78	8 P.M. — 71
3 A.M. — 68	9 A.M. — 74	3 P.M. — 79	9 P.M. — 71
4 A.M. — 68	10 A.M. — 71	4 P.M. — 79	10 P.M. — 72
5 A.M. — 69	11 A.M. — 73	5 P.M. — 77	11 P.M. — 74

TEMPERATURE, JULY 28, 1915



3. Make a graph showing the per cent of daily attendance of boys in your class or school. On the same paper make a corresponding graph for girls. For the latter use a broken line or ink of a different color.

4. Make a graph showing the changes in the average attendance for the following months:

Sept. 600	Oct. 595	Nov. 605	Dec. 595	Jan. 590
Feb. 595	Mar. 585	Apr. 565	May 570	June 560

SECTION V

ECONOMICAL BUSINESS COÖPERATION

The Market Place

In the early days of a growing community, the housekeeper depended for her supply of fresh vegetables, eggs, etc., upon the passing wagon of a farmer of the vicinity. With an increase of population, the need of a place at which a seller and a buyer could meet, developed the "market square" in the center of the village. Here, in the early morning of stated days, were assembled the wagons, from the contents of which were filled the baskets of the customers.

With a larger number of inhabitants comes a market building, in which space is provided to enable the farmer to sell his less bulky products. In the adjacent streets his wagons are permitted to stand during certain hours. Here he sells his load of hay, cordwood, etc.

The Country Store

In sections remote from a village market, the country store serves as a "produce exchange." To it are brought eggs, butter, chickens, and the like, when the quantities are too small to permit of economical consignment by the producer to a city commission merchant; and these are exchanged either for cash or for groceries, dry goods, etc. Every day or two the storekeeper can ship a box or two of eggs, a crate of chickens, to his city agent, to be sold for his account.

Written Problems

1. A farmer brings to a store 8 dozen of eggs, for which he will be allowed 15 cents per dozen in exchange for tea at 60 cents a pound, or 12 cents in exchange for calico at 8 cents a yard. What would the storekeeper gain on the transaction, if he obtained 16 cents per dozen for the eggs and gave in exchange (*a*) tea that cost him 40 cents per pound? (*b*) Calico that cost him 6 cents per yard?

2. A storekeeper paid 12¢ per dozen for 100 dozen eggs, 13½¢ per dozen for 120 dozen, and 13¢ per dozen for 80 dozen. They were sold through a commission merchant for 20 cents per dozen. What is the storekeeper's profit if the expenses for boxes, freight, and other charges amount to 12½% of the sum obtained by the commission merchant?

Coöperation in Selling

While a farmer may produce a large quantity of fruit, berries, vegetables, etc., excellent in quality, he frequently realizes only a fraction of their real value. He packs these berries badly at times, or perhaps sends them to an overstocked market. In California, Florida, Maryland, Texas, Washington, and many other states, associations have been formed to dispose of the foregoing farm products through a competent manager. The latter insists upon attractive packages, uniform quality, and he ships each article to the place at which the best net price can be obtained, as determined by daily dispatches.

A carload of California grapes started for New York is halted by telegraph at Chicago, and its destination is changed to St. Louis, if later information shows an increase of price in this city, making it a better market than New York. The manager may direct the sale at an

intermediate point of a carload of Texas peaches intended for Denver, when the quotations received by telegraph show the advisability of this procedure. The weather reports from an intended selling place may indicate the selection of a different city.

The individual farmer is taught by the manager what to raise, how to pack, etc.

Written Problems

1. A farmer shipped 1000 crates of peaches from Texas to Colorado at an expense of 27¢ per crate for picking, packing, and hauling; 40¢ for freight; 15¢ for refrigerating; and $2.60 for handling in Colorado, commission, and other charges. (*a*) What were his net receipts if the peaches sold for $3.75 per crate? (*b*) How much more or less would he receive if he had accepted an offer of 60¢ per crate for the peaches on the trees?

2. The manager of a selling association obtained $3.50 per crate for peaches at an expense of 30¢ per crate for picking, packing, etc., 25¢ for freight, 15¢ for ice, and $1.75 for commission and other charges. How much per crate was realized by the producer if his share of the association's expenses was 1% of the amount remaining after other expenses had been deducted?

Coöperation in Producing

When a farmer realizes that butter made by an expert brings a higher price than his product, he is willing to join in the organization of a butter factory ("creamery"), or at least to sell his milk to its owners. In this way he removes from the home the drudgery of churning, and obtains a greater revenue from his milk.

Written Problems

1. Find the cost of the following daily ration of a cow weighing 950 pounds, giving 20 quarts of milk per day:

Bean straw 8 lb. at $4 per ton of 2000 lb.
Mixed hay 12 lb. at $9 per ton of 2000 lb.
Wheat bran 2 lb. at $20 per ton of 2000 lb.
Corn meal 3 lb. at $2.94 per sack of 196 lb.
Cottonseed meal 2 lb. at $27 per ton of 2000 lb.
Oil meal 1 lb. at $28 per ton of 2000 lb.

2. What difference would be made in the cost of a ration in which 30 pounds of ensilage at $3 per ton are substituted for 8 pounds of hay?

3. A cow is pastured for 140 days at the rate of $1.50 per month of 30 days. To feed her for the rest of the year 30 bushels of corn and 30 bushels of oats ground together are required, together with 3 tons of clover hay. Taking the corn at 60 cents per bushel, oats at 40 cents per bushel, and the hay at $8 per ton, find the cost of the food for a year.

Coöperation with Employees

In many lines, work after the regular hours is paid for at double the regular rate.

Some factories fix a standard time for the completion of each article. An operator requiring more time is paid the regular hourly rate; one completing the daily task before the expiration of the working day receives the hourly rate for work done thereafter. The quick man thus increases his pay without diminishing that of his slower mate.

Some corporations set aside a percentage of the net profits for distribution annually among their employees in proportion to their wages.

Written Problems

1. A contractor engages to complete the electric work in a new building in 90 days. When 70 days have elapsed, he finds that it would require 25 days longer for his force of 20 men to finish the task if they continued at the regular rate of 8 hours per day. (*a*) How many hours per day must they work for the remaining time to enable their employer to carry out the terms of the contract? (*b*) What will be the daily pay of each of the employees for the remaining 20 days, at 50 cents per hour for 8 hours and double this rate for the overtime?

2. A factory makes a net profit of $50,000 per year on its capital of $250,000. After distributing 50% of the profits among the stockholders and deducting 10% of the profits to cover depreciation of the machinery, it divides the remainder among its employees whose pay roll amounts annually to $500,000. (*a*) What per cent is the salary of each increased? (*b*) How much does Mr. Dalton receive, his salary being $1200 per year?

3. If the price of articles needed by the average family has increased 10% and the expense of placing these articles in a customer's house adds 10% to their new price because of the cost of telephone messages, packing, delivery, etc., how much more must now be paid for a year's supply of goods previously obtained for $475?

4. A farmer receives 20% above the market price for butter and eggs delivered to customers who appreciate the superior quality of the articles he produces. How much more does he realize from the sale of goods that will bring $600 at the market rates, with an expense of $80, when the latter is increased 30% by the delivery of the goods at the houses of his patrons?

Price Fluctuations

The price received by the North Dakota farmer for his wheat is affected by the weather in Argentina and southern Russia. Western Europe produces much less wheat than it needs; the remainder must be procured from Canada, the United States, Russia, and Argentina, each of which has a surplus.

The rate to be paid per bushel is determined by the amount of the European shortage and the size of the available surplus. This is first reflected in the quotations from Liverpool, which has finished its transactions for the day before those of Chicago are begun, owing to the time difference of six hours.

The Exchanges

Each city of any size has one or more exchanges; a produce exchange, a cotton exchange, a metal exchange, a tobacco exchange, a stock exchange, etc., either singly, or in combination, as a Board of Trade, for instance.

At each is received a continual stream of telegraphic information concerning the movements of commodities, weather conditions, rates of freight, etc., each of which has some effect on prices. The general effect of the exchange is beneficial, although some persons use it as a means of speculation.

An increase in a local price is reduced by the prompt receipt of a supply from a neighboring section when the cost at the latter is sufficiently less to stand the cost of transportation.

Cabbage and eggs are frequently shipped from Germany to our Atlantic ports, as are potatoes from Scotland and from Ireland. Australia sends meat to California.

Sight Problems

1. Find the brokerage on each of the following:

   a. 4000 bushels of oats at $\frac{1}{8}\,¢$ per bushel.
   b. 320 shares of stock at $12\frac{1}{2}\,¢$ per share.
   c. 240 bags of coffee at $4\,¢$ per bag.
   d. 1000 barrels of pork at $2\frac{1}{2}\,¢$ per bbl.
   e. 1500 bales of cotton at $5\,¢$ per bale.

2. A broker received a commission of $150 on a sale of wheat. How many bushels did he sell if his commission was $\frac{1}{8}\,¢$ per bushel?

3. At the rate of 4 cents per bag, how many bags of coffee must a broker buy to entitle him to a commission of $100?

4. How much does a broker receive for selling a $10,000 bond at the rate of $\frac{1}{8}\,\%$?

Written Problems

1. A broker buys for the Planet Mills 10,000 bushels of wheat at $1.05 per bushel. What is the cost delivered at the mill, including brokerage of $\frac{1}{8}\,¢$ per bushel, and $\frac{1}{8}\,¢$ per bushel for transportation charges?

2. Find the cost of 1000 barrels of pork at $21.75 per barrel and brokerage at $2\frac{1}{2}$ cents per barrel.

3. The Fulton Mills bought 1000 bags of coffee, weighing 130 pounds each, at $12.42\,¢$ per pound and paid $4\,¢$ per bag brokerage. (*a*) Find the cost of the coffee. (*b*) What will be the cost of the coffee when it is roasted, if the expense of roasting is $\frac{1}{2}\,¢$ per pound? (*c*) If it is then put into bags holding 100 pounds each, how many bags of roasted coffee will there be, if the coffee loses 15 % of its weight by the roasting?

Sharing Expenses

Preparatory Exercises

1. Mr. A needs the use of a traction engine for 120 days in a year. Mr. B needs it for 100 days, and Mr. C for 80 days. (*a*) For how many days do the three need it? (*b*) For what part of this time does Mr. A need it? (*c*) Mr. B? (*d*) Mr. C?

2. If the three unite in the purchase of an engine for $1500, with the understanding that Mr. A should have the use of it for $\frac{2}{5}$ of the year, Mr. B for $\frac{1}{3}$ of the year, and Mr. C for $\frac{4}{15}$ of the year, how much should (*a*) Mr. A contribute to the cost? (*b*) Mr. B? (*c*) Mr. C?

3. Mr. Miller insures his property in Company No. 1 for $2000 and in Company No. 2 for $4000. What share of a loss should be paid (*a*) by No. 1? (*b*) By No. 2?

4. If Mr. Miller's loss by fire is $1200, how much should he receive (*a*) from Company No. 1? (*b*) From Company No 2?

Written Problems

1. A fire loss of $3915 is shared by three companies, company A having insured the property for $4000, B for $6000, and C for $8000. What share of the loss should be paid by each?

PROCESS

$4000 + $6000 + $8000 = $18000. Total insurance.
18000 : 4000 : : 3915 : A's share
18000 : 6000 : : 3915 : B's share Test by adding the
18000 : 8000 : : 3915 : C's share three shares.

2. In a storm it was necessary to throw overboard 90 tons of wheat in order to save a vessel. If the cargo consisted of 105 tons belonging to A, 180 belonging to B, and 315 belonging to C, (*a*) what share of the loss should each bear? (*b*) How many tons of the remaining cargo should each receive?

3. A fertilizer contains 10 parts of nitrate of soda, 17 of kainit, and 53 of phosphate. How many pounds of each are contained in a ton?

4. A farmer needs 2700 cu. yd. of concrete. How many cu. yd. (*a*) of cement, (*b*) of broken stone, and (*c*) of cement must he procure if 1 cu. yd. of the first is needed to $1\frac{1}{2}$ cu. yd. of the second and 2 cu. yd. of the third?

Partnership

Wm. Arnold and John Fleming were employed by the same firm at a monthly salary of $100 each. The former had saved $3000 and the latter $2000. They started in business as Arnold and Fleming, each contributing his entire savings to the capital of the new firm. The written agreement provided that the profits should first be devoted to the payment of 6% interest to each on the sum contributed by him, and that the remainder, if any, should be equally divided between the two partners.

Sight Problems

1. What is the yearly interest at 6% on $5000?

2. What remains from profits of $3300 after the deduction of the above interest?

3. What is (*a*) Mr. Arnold's share of the profits, including interest on the capital invested by him? (*b*) Mr. Fleming's share?

Written Problems

1. The year before going into business, Mr. Arnold and Mr. Fleming received 4 % interest on their savings. How much was the income of each increased during the next year?

2. What would be the share of each in the profits, if he was allowed $1200 for his services, and the remainder was divided in proportion to the respective investments?

3. If each left in the business $1000 of his profits, how would profits of $3920 be divided at the end of the succeeding year on the basis of the original agreement and taking into consideration the new capital?

Bankruptcy

When a business man (or a firm) finds that he is unable to pay all of his debts, he notifies his creditors, giving them a statement of his *assets* and his *liabilities*, the former including the goods in stock, money due him, etc., and the latter being the money he owes. The creditors may agree to accept a per cent of their respective claims in full settlement, taking a portion of the sum in cash, and the remainder in notes payable at stated intervals. In this way he may continue in business, the profits enabling him to meet the notes as they fall due.

If the creditors are not satisfied with any terms of settlement proposed by him, they may procure an order of a court for the sale of the stock, etc., also other property owned by any of the partners, and the distribution of the proceeds among them in proportion to their respective claims.

Preparatory Exercises

1. M's creditors accept his offer of 70 cents on the dollar. How much does A receive, to whom M owes $500?

2. A's stock, accounts, etc., realized $1200 above the expenses of bankruptcy proceedings, auctioneer's fees, etc. If his debts are $2000, what per cent of them can be paid?

Written Problems

1. Alphin & Co. failed with liabilities of $7240 and assets that were sold for $4525 above expenses. What per cent can be paid each creditor?

2. At the time of the failure of Martin E. Lynch he had four creditors, to whom were due the following:

Mr. A, $500; Mr. B, $600; Mr. C, $700; Mr. D, $800.

His assets realize $2000. What does each creditor receive?

3. Frank Carroll's creditors accept his offer of 20 % in cash, 20 % in 3 months, 20 % in 6 months, and 20 % in 9 months with interest at 5 % on the deferred payments. How much does he pay Gerald Tofte, including interest, to whom he owes $1200, if he makes the foregoing payments according to agreement?

4. Although freed from liability for the remaining 20 %, he pays one half of it one year from the date of failure, and the remainder one year later, with interest at 6 %. What is the amount of these two payments?

5. The books of a concern showed assets of $25,000 and liabilities of $30,000. If the former realized only 66 % of their book value, (*a*) what per cent of his indebtedness can be paid each creditor? (*b*) How much will John Rodman receive, to whom the company owes $375?

Corporations

Believing that a trolley line between Fairfax and Cameron would not only increase the value of their property but would also yield a fair return upon the money invested, a number of the residents agree to organize the Lorton Valley Railway Company. Having ascertained that the line can be built and equipped for $90,000, the *capital stock* is fixed at $100,000, to consist of 1000 *shares* of $100 each.

A dozen men pay up $100 per share to the amount of, say, $5000, and make application to the legislature for a *charter*, giving the name of the company, the names of the incorporators, with the number of shares held by each, and the amount paid therefor in *cash*.

Stockholders. — When the charter is granted, the remaining stock is offered for sale. Each purchaser receives a certificate specifying the number of shares he holds.

Officers. — When sufficient money is obtained from the sales of stock to warrant building operations, the organization of the company is completed by the election of a president, a vice president, a secretary, a treasurer, and a board of, say, twelve directors, each stockholder having one vote for each share of stock owned by him.

Dividends. — At the end of the first year of operation, the profits of the company are found to be $6716.25. To keep the equipment in proper condition, etc., the directors agree to distribute only $4500 among the stockholders as *dividends*, retaining $2216.25 as a *surplus*, to be used as occasion may require. This is added to any portion of the $100,000 not already expended and any other surplus. A dividend of 4½ % is then *declared*, and a check for the sum to which he is entitled is mailed to each stockholder.

Preparatory Exercises

1. A withdraws $1000 from his savings bank and buys 10 shares of the stock. To what fraction of the profits is he entitled?

2. If the profits distributed among the stockholders the first year are $4500, (*a*) how much does A receive? (*b*) What per cent does he receive on his investment?

3. How much more does A receive on the investment than he would have obtained from the bank, which paid 4 % interest?

4. A sells his stock to B at $125 per share. If the latter receives $5 per share from the profits, (*a*) what per cent does he receive on his investment? (*b*) What per cent of the original price paid by A does the latter receive from B?

5. If the profits of the railway company for a year are $7000, (*a*) what surplus will remain after a 5 % dividend is paid? (*b*) What per cent is earned on the capital?

6. When the holder of $100 shares receives dividends of $7 per share, how many dollars per share can a person pay for them to obtain 5 % on the cost?

7. If a woman pays $150 per share and receives dividends of 9 % of the face value ($100) of the share, what per cent of the price paid per share does she receive in dividends?

The Broker. — When the investor, after consultation with a thoroughly reliable and disinterested person, selects the stock he desires to purchase, he may have to employ a broker to obtain it for him, who is paid $\frac{1}{8}$ or $\frac{1}{4}$ per $100 share for his services, which include the transfer of the stock to the buyer.

Sight Problems

1. What does the buyer of 100 shares of stock pay for it at $\$139\frac{1}{2}$ per share and a brokerage of $\$\frac{1}{8}$ per share?

2. What does the seller of 100 shares of stock receive for it at $\$129\frac{1}{2}$ per share less the brokerage of $\$\frac{1}{8}$ per share?

3. What per cent does a man obtain on stock costing \$150 per share which gives an annual dividend of \$6 per share?

4. What can a man afford to pay per share for stock that gives an annual dividend of \$9 per share, if he is satisfied with 4% on the money invested?

Written Exercises

1. Find the cost of 150 shares of stock bought at $\$143\frac{3}{4}$, brokerage $\frac{1}{8}$.

$$\$(143\tfrac{3}{4} + \tfrac{1}{8}) \times 150.$$

2. How much does Mr. Watson receive for 150 shares of stock sold at $\$143\frac{3}{4}$, brokerage $\frac{1}{8}$?

$$\$(143\tfrac{3}{4} - \tfrac{1}{8}) \times 150.$$

3. How many shares of stock at 80 can be bought for \$9615, brokerage $\frac{1}{8}$?

$$\$9615 \div \$80\tfrac{1}{8}.$$

4. Mr. Romeo realized \$17,253 from the sale of stock at 80. If the broker charged $\frac{1}{8}$ for selling, how many shares were sold?

5. When semiannual dividends of $4\frac{1}{2}\%$ are received from stock bought at 175, including brokerage, what is the annual rate of income on the investment?

$$9\% \div 1.75.$$

Increasing Capital Stock. — A trolley company that has a capital of $100,000, on which it pays regular dividends of 6%, may consider it advisable to expend $50,000 in the extension of the road, etc. This additional money may be obtained by the issue of five hundred additional shares of stock, each of the existing stockholders being entitled to subscribe at par (face value) for one share for each two of the other stock owned by him.

A stockholder not desirous of increasing his holdings, or being without the necessary funds, could dispose of his "rights," obtaining therefor $10 per share from a person willing to invest in the new stock. As the latter gets it at par from the company, it costs him $110 per share, including the price paid for the "right."

Common and Preferred Stock. — Sometimes a corporation decides to make the new issue *preferred stock*, by using the earnings in the first instance to pay a dividend of, say, 5% on it before paying any dividend to the owners of the *common stock*.

This guarantee is an inducement to investors to pay as much as, say, $115 per share for preferred stock.

Assume that the trolley company's net earnings for the first year after the issue of $50,000 preferred stock are $6500. From this must be deducted $2500 dividend on the latter, leaving $4000 for the holders of the common stock, or a dividend of 4%. At this time, the preferred stock may bring $110 per share and the other $90 per share. When the earnings increase to $8500, leaving $6000 for distribution among the holders of the common stock, the latter may go to, possibly, $140.

Some corporations make another issue of preferred stock, which is called *second preferred*, the previous one being the *first preferred*.

Buying and Selling Stocks. — The small investor can seldom afford to risk his savings, and he therefore limits his investments to stocks of corporations that have paid regular dividends for a number of years. Companies are organized every day merely to unload worthless stocks upon foolish buyers, employing for the purpose unscrupulous agents at a large commission. Much of the nominal capital of, say, $100,000 goes into the pockets of the latter; some of it is wasted in the excessive prices paid to the organizers or their friends. The declaration of a dividend of six or seven per cent does not mean that it is warranted by the profits.

Bonds

Instead of increasing the capital to $150,000, the company may conclude to issue bonds for $50,000, bearing interest at, say, 5%. As security, the company mortgages the property in favor of the bondholders.

A person that would not risk his money in stock may be willing to invest in the bonds, owing to the security obtained through the mortgage.

A corporation first pays interest on its bonds, then dividends on preferred stock, then dividends on common stock.

The price of a bond is stated as a per cent of the face value, 109½ meaning that a $100 bond would cost $109.50, exclusive of brokerage; a $10 bond, $10.95; a $500 bond, $547.50, etc. The brokerage of ⅛ means $⅛ for each $100 in the face of the bond, regardless of its cost.

A *registered* bond is made out in the name of the buyer and the interest check is mailed to his residence. A *coupon* bond contains a certificate for each interest installment, which is cut off when the interest is due and presented at the company's office for payment.

Sight Problems

1. What is the annual interest on bonds for \$10,000 at 6%?

2. What is the cost of \$10,000 in bonds at $137\frac{3}{4}$?

3. What is the brokerage on \$10,000 in bonds at $\frac{1}{8}$%?

4. How many \$1000 4% bonds must be purchased to yield \$1000 annual interest?

5. What would be the cost of 4% bonds yielding \$1000 interest annually when the bonds are selling for 96, including brokerage?

Written Problems

1. An electric-light company pays 5% interest on its bond issue of \$100,000, 6% dividends on \$50,000 preferred stock, and 7% dividends on \$100,000 common stock. What were the net earnings for the year if there remained a surplus of \$3745.60 after the foregoing payments were made?

2. A man wishes to invest a sum that will furnish an income of \$1500 per year. (*a*) What will be the face value of 5% bonds that will yield \$1500? (*b*) What is the cost of the bonds at $109\frac{7}{8}$, and $\frac{1}{8}$ brokerage?

3. How much does the Home Trust Co. pay for 25 one-thousand-dollar bonds at $104\frac{3}{8}$, brokerage $\frac{1}{8}$?

4. What is the annual income from \$21,750 invested in 5% bonds bought at $108\frac{5}{8}$, brokerage $\frac{1}{8}$?

5. A trolley line has a capital of 1000 shares, par value \$100. It has issued and sold 5% bonds of the face value of \$100,000. If the net earnings for the year are \$11,750, what will be the surplus after paying interest on the bonds and a 6% dividend on the stock?

Accrued Interest

Mr. Ziegler is willing on July 2 to pay \$120 per share for stock in the Happy Valley Irrigation Company, which pays regular semiannual dividends of \$2½ on Jan. 1 and July 1. On October 1 the price might be \$121¼ to give the seller the earnings of the three months that elapsed since he received a dividend, the buyer getting the new one in three months.

While the price of the 5% bonds of the same company might remain at 120, Mr. Ziegler would have to pay the seller the interest that *accrued* from the last interest date. If he bought a \$100 bond on October 1, at 120, the cost would be \$120 plus the interest at 5% on \$100 from July 1.

Written Problems

1. (*a*) What does Mr. Ziegler pay for the foregoing bond? (*b*) Find the cost of a bond for \$10,000 bought at 120, Dec. 1, brokerage ⅛, and accrued interest.

2. Mr. Bush has a \$1000 six per cent bond, interest payable March 1 and Sept. 1. (*a*) How many days are there between these dates? (*b*) What fraction of the half-yearly interest due Sept. 1 has accrued on June 24? (*c*) If Mr. Bush sells the bond at 120 and accrued interest, how much does he receive for it, no brokerage? (*d*) If Mr. Bush cuts off and retains the September coupon, how much should he receive for the bond?

3. The owner of a farm of 242 acres sold it at \$160 per acre. He invested one half of the proceeds in 4% bonds at 88, including brokerage, and the remainder in 5% bonds at 110, including brokerage. (*a*) What is the par value of the 4% bonds? (*b*) Of the 5% bonds? (*c*) What annual income is yielded by the bonds?

Maturity of a Bond. Bonds are issued for various terms up to, say, 50 years. At that time interest ceases, and they are redeemable at their face value.

A person purchasing on the day of its issue a 50-year 5% bond of $100 would receive in interest to the time of maturity $250. If he paid $120 for it, he would lose $20 at its redemption for $100, making his net income, at simple interest for 50 years, $230, which is $4.60 per year. As the bond cost $120, the rate received is 4.60 ÷ 120, or $3\frac{5}{6}$%, at simple interest.

After the day of issue these bonds must be sold at a lower price to enable a purchaser to realize $3\frac{5}{6}$%.

Written Problems

1. A 7% bond for $1000 has 10 years to run. (*a*) How much does it yield in that time at simple interest? (*b*) What is the total sum received in 10 years, including the face of the bond? (*c*) How much more than $1200, the cost of the bond, is received in 10 years? (*d*) What does this average per year? (*e*) What % of $1200 is this average?

2. A 3% bond for $100 maturing in 10 years was bought for $90. (*a*) How much interest is received in 10 years? (*b*) How much is received, including interest and the face of the bond? (*c*) How much more than the cost of the bond? (*d*) What is the average per year? (*e*) What % of $90 is this average?

In determining the rate paid by a bond, bankers take into consideration the interest they can earn upon each dividend (compound interest). A bond paying interest semiannually would show a higher rate than one paying interest annually.

Safeguarding our Business Interests

Insurance

The chief business of the head of a household is the care of those dependent upon him. This requires the continued setting aside of a portion of the income for the time when it may be interrupted by the producer's illness or terminated by his death. Provision must also be made for increased expenses of various kinds.

Personal Insurance

In many instances, the meager savings of an individual are insufficient to meet his emergency, while the combined small contributions of a number can be so managed as to provide a measure of relief for the comparatively small number that require financial help.

Fraternal Societies

Organizations of various kinds pay a fixed weekly stipend for a stated period to each member unable to work because of illness, and also a definite sum to his family at his death.

Insurance Companies

Insurance companies provide three forms of personal insurance: health insurance, accident insurance, and life insurance. The first gives the insured person a weekly sum for about one half year to each member incapacitated from work by ordinary illness. The second also provides a weekly payment, but only in case the incapacitation is caused by an accidental injury, and also a definite sum when death results therefrom during the period covered by the insurance.

Life Insurance

The most common form of personal insurance is one calling for the payment of a definite amount on the death of the person insured, in consideration of the payment of a stated sum annually, called the *premium*.

The terms of the contract are set forth in a document called the *policy*, which specifies the sum payable by the company upon proof of the death of the *assured*, together with the annual *premium* payable by the latter, and the date on which it is due.

In the *ordinary life policy*, the premium payments continue throughout the life of the assured. This form furnishes the largest insurance for a given premium. A person that fears his ability to keep up payments after his capacity to earn has seriously diminished takes out a 20-payment life policy. In this form premium payments cease after 20 years. A 10-payment life policy is preferred by those whose increase in expenses is likely to be greater than their increase in earnings, and who, therefore, desire to complete their payments within the stated period.

Investment and Insurance

A favorite policy with those who can afford the greater cost is the *endowment policy*. This provides for the payment of the face of the policy by the company at the end of the term, or upon the death of the assured at any time during the term. These policies run, as a rule, for ten or for twenty years.

If, at the end of the term, the assured has no special need of the money, he can purchase therewith a paid-up policy for a stated amount.

The following table shows the premiums payable, at the specified ages, per $1000 of insurance in the ordinary life plan, the 20-payment life, the 10-payment life, and the 20-year endowment.

ANNUAL PREMIUMS PER $1000

AGE	PREMIUMS PAYABLE FOR			
	Life	20 Years	10 Years	20 Years
20	$18.01	$27.78	$45.51	$47.54
25	20.14	30.07	49.11	48.03
30	22.85	32.83	53.38	48.71
35	26.35	36.17	58.44	49.75
40	30.94	40.34	64.14	51.39
45	37.09	45.69	71.66	54.15
50	45.45	52.83	80.51	58.76
55	56.93	62.66	91.42	66.32
Policy payable at death				In 20 years

Premiums and Dividends

From figures collected for a long period and covering a very large number of people of all ages, it is possible to determine the average number of years persons of a given age will live.

From this is calculated the sum that must be invested annually to yield with interest a given amount, say $1000. To this is added a certain per cent thereof to cover expenses, and an additional per cent to provide for any unusual mortality caused by pestilence, flood, etc. From time to time the portion of the excess premium that can be returned to the assured is determined. This so-called *dividend* is either paid him in cash or used to increase the amount payable at his death.

Insurance on Property

Insurance companies of different kinds afford all classes of persons the opportunity to avoid a financial loss that might seriously cripple them. *Fidelity insurance* secures employers against loss by thefts of employees; *burglary insurance* reimburses the owner of valuable articles stolen from his premises; *plate-glass insurance* replaces costly store windows that have been broken; *casualty insurance* assures for the owner of buildings any liability for damage because of injuries occurring on his property.

Fire Insurance

Insurance against fire is needed by every householder. The expense is so insignificant compared with the cost of replacing the building or the furniture destroyed that no one is justified in neglecting to insure his property.

The rate of insurance is determined by the risk. It is greater on a frame building than on one of brick or stone. It is smaller on a detached building than on one located in a row. The nature of the business carried on in a building is a factor in fixing the rate, one in which gasoline, for instance, is sold, being very hazardous. The protection against fire is another factor, a poor water supply in one section of a city increasing the rate above that charged in sections in which the water supply is better.

Since an insurance company may be bankrupted by a local fire that destroys hundreds of houses, a business man generally distributes his insurance among several companies, each located in a different city. For convenience, he "places" his insurance through a broker, who is paid a commission by such company.

The following is the bill of an insurance agent:

CHICAGO, ILL., October 1, 1915

MR. SAMUEL PRATT
819 Wabash Av.

To HENRY COLLINS & SON, *Dr.*
Insurance Agents and Brokers
LaSalle Building

		To premiums on the following policies:		Amount	Premiums	
Oct.	*1*	*Phœnix of New York*	*Stock*	*1500*	*11*	*25*
	3	*Fireman's of Denver*	*Bldg.*	*9000*	*54*	*00*
		Fidelity of Hartford	"	*3500*	*21*	*00*
	5	*Northern of London*	*Stock*	*3000*	*22*	*50*
					$108	*75*

Even the small insurer finds it advantageous to obtain his insurance through a reliable broker, as the latter keeps him informed of the date of the expiration of each policy, the standing of companies, etc., without charge.

The indorsement of a policy of fire insurance shows (*a*) the date of the expiration of the insurance, (*b*) the location of the property, (*c*) the sum for which the company insures it, (*d*) the premium, (*e*) the name of the owner of the property, and (*f*) the name of the insurance company.

The most prominent feature in the indorsement of the policy is the date of its expiration, which should always be kept in mind.

In the case of a building injured by fire, the insurance company has the right to make the repairs needed to restore the building to its condition before the fire, instead of giving the insurance money to the owner. An itemized statement of the value of the goods injured by fire must be supplied by the owner within a definite time and an opportunity given the company to examine their condition, etc. The inventory, from which the details can be supplied, is shown on page 63.

Rates for Short Terms

The owner of a building seldom insures it for less than a year, and generally for three years because of the reduction.

A farmer who intends to sell his hay, grain, etc., can insure it for 2 days, 5 days, 1 month, etc.

The accompanying table gives the per cent of the annual rate charged for each period specified:

2 days 3%	15 days 14%	3 months 40%
5 days 7%	20 days 15%	6 months 70%
10 days 10%	1 month 20%	9 months 85%

A complete table gives the rates for other periods.

Written Exercises

1. What annual premium must Mr. Wilkes pay on a 20-year endowment policy if his age is 30 years?

NOTE. — See table, p. 278.

2. Andrew Jenkins takes out a life policy for $2000 at the age of 30. If he dies after paying 30 annual premiums, (*a*) how much has he paid the company? (*b*) How much less would he have paid if he had taken out a policy on the 20-payment life plan? (*c*) On the 10-payment life plan?

3. Mr. Wade has property worth $6500. He insures it for 80% of its value. (*a*) What is the face of the policy? (*b*) What is the annual premium at 80¢ per $100? (*c*) What must he pay to insure it for three years, the rate being twice that for one year's insurance?

4. A dealer buys 12,000 bushels of wheat at 95 cents per bushel. What must he pay to insure its cost for 10 days at 10% of the annual rate of 50 cents per $100?

Governmental Helps

Among the agencies that help us in the conduct of our business, the most important are the municipal, the county, the state, and the national government. These coöperate by educating the children, by constructing roads and streets, by protecting property against fire and theft, by affording facilities for recreation, by furnishing relief for the poor, by handling our mails, by increasing the commercial advantages of rivers and harbors, by regulating railroads and other public corporations, etc.

Governmental Revenues — Taxes on Persons

Money to pay the expenses of a government is obtained from various sources. A portion of it comes from *license fees* paid by peddlers, storekeepers, owners of vehicles, proprietors of shows, chauffeurs, etc.

In some places a *poll tax* of fifty cents, a dollar, etc., is collected from every male resident that has reached the age of twenty-one.

In many states an *inheritance tax* is levied on the property in the state owned by a person at the time of his death.

An *income tax* is collected by the national government from all persons whose income exceeds a certain sum.

Governmental Debts

When the cost of a much-needed improvement is so great that the taxes for the year in which it is made should not be burdened with the entire expense, money is borrowed, and the cost spread over a number of years. Thus, a portion of the expense of erecting a new school is paid annually for perhaps ten years or longer.

How Tax Money is Spent

The following shows the distribution of the governmental expenditures of Johnson Co., including the city of Blissville:

Interest on Debt, etc.	$ 51,255.17
General administration	3,328.00
Judicial and Correctional	9,870.26
Educational	36,116.31
Health and Sanitation	17,076.18
Protection of Life and Property	30,179.61
Maintenance of Highways, Bridges, etc.	8,053.46
Parks, Public Buildings, etc.	2,373.57
Advertising, Printing, etc.	1,354.00
State Taxes	4,301.65
Charitable Purposes	10,309.38
	$174,217.59
For deficiencies in collections	3,782.41
	$178,000.00
Distributed as follows:	
City Charges	$160,873.43
County Charges	9,824.92
State Taxes	4,301.65
	$175,000.00

Assessed value of the property of Johnson County was:

Real Estate	$12,840,500
Personal Property	1,659,500
Total	$14,500,000

Sum to be raised by taxation:

Expenses during 1915	$175,000.00
Less estimated revenues	24,492.90
	$150,507.10

At 12:34 P.M., Sept. 6, 1914, the taxes payable on property become a *lien* thereon, and a buyer after that time deducts from the purchase price the taxes then due, although he is not required to pay them until Oct. 30.

Assessed Value of Property

Officials called *assessors* fix the valuation of property for purposes of taxation. As the owner of real estate is frequently unable to obtain more than 60 or 70 per cent of its actual value, it is customary to assess city property at, perhaps, two thirds of its value, and farm lands still lower. The ratio the *assessed value* bears to the actual value does not affect the taxes on a particular parcel provided the rate is uniform throughout the district in which the taxes are to levied.

Assuming that the property in a district is worth $8,000,000 and the taxes to be raised are $40,000, A, who owns property worth $8000, should pay $40, or $\frac{1}{2}\%$ of its actual value. If the assessed value is made $4,000,000, the tax rate is 1%, but A's being assessed for $4000, his bill at 1% is still $40.

Real and Personal Estate

Many owners of personal property escape the payment of their share of governmental expenses because of the difficulty of ascertaining their possessions. In the valuation given on the preceding page, that of the personal property is possibly less than one tenth of its real worth.

The Taxpayer

Everybody pays taxes either directly or indirectly. When he is not the owner of the house in which he lives, the rent he pays includes the taxes paid by the owner. His interest in the proper expenditure of every cent raised by taxation is just as great as that of the person who gets a tax bill and pays it.

Assessments

While every property owner in a given district is charged the same rate for the general expenses, the cost of certain local improvements is paid by the persons immediately benefited by them. Thus, when a city constructs a sidewalk, say, 800 feet in length, the owner of 200 feet of property facing the street pays one fourth of the cost. This is generally called an *assessment*, and is not made a part of the regular taxes. The construction of a short road used chiefly by the owners of the adjacent farms is charged against them.

State Taxes

Money required for state expenditures in excess of the revenues is collected through a tax, which is included in the county tax bills. The cost of a long highway may be paid partly by the state, partly by the counties through which it passes, and partly by the owners of abutting property.

Written Exercises

1. (*a*) Find the rate on $1 (correct to seven decimal places) to yield $150,507.10 taxes on property assessed for $14,500,000.

(*b*) What are A's taxes on a house assessed for $3500?

2. Oct. 1, 1914, Frank Phillips agrees to pay Mr. Collins $5000 for his house. How much should he pay after deducting the taxes, the house being assessed at 70% of its value of $5000, and the tax rate being $10.3798 per $1000 of the assessed value?

3. Find the taxes on the property covered by the following bill.

A TAX BILL

CITY OF BLISSVILLE — DEPARTMENT OF FINANCE

Taxes for 1914

Draw check to the order of Receiver of Taxes

Rate 1.03798 per cent Penalty takes effect Nov. 1

SECTION	VOL.	BLOCK NO.	LOT NO.	LOCATION	VALUATION	TAX
18	4	6036	30	23d St.	$3000	
				Interest on Tax		$
				Rebate on Tax		

4. In Blissville taxes are received at any time after Sept. 1, a rebate being allowed for payments made during this month at the rate of 6 % per annum for the time between the date of payment and Nov. 1. What rebate is allowed on the foregoing bill if it is paid Sept. 2?

5. Taxes paid during October receive no rebate. Those paid after Oct. 31 are subject to a penalty of 7 % per annum from Oct. 1. (*a*) What are Mr. O'Donnell's taxes on property assessed for $10,000 when paid Oct. 30, 1914, the rate being 1.03798 %? (*b*) What amount is payable Jan. 2, 1915, if interest at 7 % is added from Oct. 1? (*c*) How much could he have saved by borrowing the money from a bank at 6 % Oct. 30, and repaying it Jan. 2? (360 days to the year.)

6. A town needs to raise $18,950 by taxation. There are 600 persons liable to a poll tax of $1.50 each. The assessed value of the property is $1,900,000. (*a*) How much is raised by the poll tax? (*b*) How much must be obtained by property taxes? (*c*) What must be the tax rate on each $1000 of valuation?

Business Communications

The government takes a letter from a box in which it is deposited and insures its delivery to the addressee in any part of the civilized world. This is done at a nominal cost through the coöperation of the mail wagon, the railroad, the ocean steamer, the letter carrier, etc.

Without leaving his desk, the business man can communicate with a customer in Europe, Asia, etc., telephoning his message to the telegraph office. The reply is telephoned him, if he so desires.

Domestic messages are charged a minimum rate for 10 words, or less, with an additional charge per each word in excess of 10, the name of the sender and the name and address of the person to whom it is sent not being counted.

These rates are usually stated as 25 and 2, 50 and 4, etc., meaning 25¢ (or 50¢) for the first ten words and 2¢ (or 4¢) for each additional word.

Sight Exercises

1. At 50 and 4 what is the cost (*a*) of a 15-word message? (*b*) Of a 30-word message?

2. Give the cost at 25 and 2 of (*a*) a 15-word message. (*b*) A 30-word message.

Day Letters — Night Letters

To secure a fuller use of their lines, telegraph companies accept 50-word messages at reduced rates subject to slight delay in delivery. A *night letter* of 50 words is sent at the regular rate for a 10-word message, and a *day letter* at $1\frac{1}{2}$ times this rate. The latter is delivered the day it is filed, and the former the next morning.

Written Exercises

1. On a telegraph blank write a message to a friend stating that you intend visiting his city. Give the route you will take, state the time you will reach his city, also the address of the place at which you will stop while there. Endeavor to make it as brief as possible by the omission of unnecessary words, but taking care to convey all necessary information.

2. Write on the proper blank a 50-word night letter to a commission merchant stating that you are now digging your potatoes and desire to know the condition of the market in his city. Ask his advice as to the quantity you should send, the time of shipment, etc.

Cable Codes

The high cost of a direct cable message, in which the names and the addresses are counted, makes it necessary for business men to use a *code* in which each word represents a group.

The following are taken from a travelers' cable code distributed gratuitously by a company:

Amble. Arrived here, all well, leaving for home at once.
Askew. Advise you by all means to have a consultation of physicians.
Bayou. Business is quiet; everything all right and everybody well.
Giant. Cannot answer positively at present. Will write.
Punch. Please accept my heartiest congratulations.
State. Please open credit in my favor through . . . for sum of . . . . Wire to me when it is opened, as I wish to draw against it at once.

A code word filed with the company by each business house gives the name and address of the latter.

Cable Letters — Week-end Letters

A 12-word *cable letter* in plain language can be sent to London or to Liverpool at a reduced rate, to be mailed to any part of Europe without an additional fee, or to be delivered by telegraph the next day at a given low rate.

A 24-word *week-end letter* is received Saturday to be delivered by telegraph anywhere on Monday morning or to be sent by mail from London.

Written Exercises

1. Find the cost (*a*) of a 6-word direct message from Tucson, Arizona, to Vienna, Austria, at 44 cents per word. (*b*) Of a 12-word cable letter at the rate of $1.75 to London and 7 cents per word additional to Vienna. (*c*) Of a 24-word week-end letter at $2.15 plus 9 cents per word. (*d*) Of a 24-word dispatch from Boise City to Calcutta at 12 cents per word from Idaho to New York and 74 cents per word from New York to India.

Standard Time

A west-bound traveler passing through a town noticed children issuing from the school and employees from the factory. His watch indicated 12 : 30, while the station clock showed 11 : 30. Upon inquiring, he ascertained that it was just noon in the town, but that the station agent used *central standard time*, while his own watch was set to *eastern standard time*.

He then learned of the coöperation of the railroads in establishing a uniform time throughout each of the four belts into which the United States is divided for this purpose, and making the time difference one hour between any two adjoining belts.

Time Belts.— The belts are approximately 15° in width, each extending about 7½° on both sides of the meridians of Philadelphia (75°), Memphis (90°), Denver (105°), and Virginia City (120°), respectively, which use Eastern (E. T.), Central (C. T.), Mountain (M. T.), and Pacific (P. T.) Time, respectively.

Changing the Time.— The point at which a railroad changes from one time to another is stated on its time-tables. The Santa Fé uses C. T. between Chicago and Dodge City, M. T. between Dodge City and Seligman, and P. T. west of Seligman.

The time used in a city near the dividing line is fixed by the local authorities, being generally that of the railroad passing through it. When a town in longitude 112½° began the use of standard time by putting forward its clocks from 11 : 30 to 12, the residents changed the time of commencing dinner from 12 to 12 : 30. When a town in the same longitude a few miles north or south, but on a different railroad, set back its clocks from 12 : 30 to 12, it fixed the time for the beginning of the midday recess at 11 : 30.

Sight Problems

1. A baseball match in Chicago (C. T.) is finished at 5 P.M. Allowing 5 minutes for transmission, what time does the score reach a newspaper office (*a*) in New York (E. T.)? (*b*) In San Francisco (P. T.)? In Denver (M. T.)?

2. A train leaves at 9 A.M. C. T., due to arrive at its destination in 24 hours. At what hour should it arrive when the latter is located (*a*) in a section having M. T. ? (*b*) In one having P. T. ? (*c*) In one having C. T. ?

3. A train leaves at 8 A.M. (C. T.) and arrives at 9 P.M. (E. T.). What time is taken for the trip ?

Standard Time in Other Countries

Practically all civilized countries have standard time. When the extent of the country east and west is very great, as in the case of the United States, Canada, and Australia, belts are employed, Canada having five, the most easterly using Atlantic Time (A. T.), which is that of Cape Breton (longitude 60°).

As an additional step towards simplification, the time of the meridian of Greenwich (longitude 0°) has been adopted by many other countries as the base, the time of each of these countries being the same as that of England or an exact number of hours earlier or later.

France, Algeria, Tunis, Spain, Belgium, Brazil, etc., use Greenwich time throughout; Germany, Denmark, Norway, Sweden, Italy, etc., use time 1 hour later; Chile uses Eastern Time; etc.

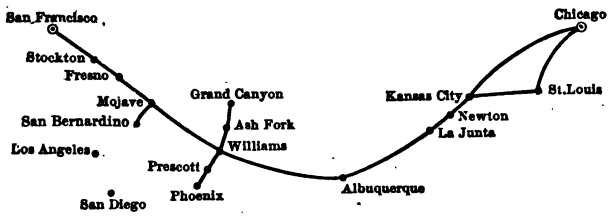
Sight Problems

1. When a message is telegraphed from London at noon, at what time, allowing ten minutes for its transmission, does it reach (*a*) Berlin? (*b*) Chicago? (*c*) Rio de Janeiro? (*d*) Santiago? (*e*) San Francisco?

2. Allowing 15 minutes for transmission, give the time at which messages telegraphed from Denver at noon will reach (*a*) London. (*b*) Paris. (*c*) Chicago. (*d*) San Francisco. (*e*) Cape Breton.

3. A vessel leaves New York Monday noon for Liverpool. Another vessel leaves Liverpool Monday noon for New York. Each makes the trip in 6 days of 24 hours each. At what hour Sunday does (*a*) the first reach Liverpool? (*b*) The other reach New York?

Railroad Travel



CHICAGO, KANSAS CITY, AND CALIFORNIA

READ DOWN				READ UP	
1	3	Miles	Stations	4	2
10.10 P.M.	8.05 P.M.	0	Lv. (C.T.) Chicago Ar.	11.00 A.M.	10.00 A.M.
11.00 A.M.	8.50 A.M.	454	Ar. Kansas City Lv.	10.30 P.M.	9.00 P.M.
	11.30 P.M.	—	Lv. St. Louis Ar.	7.43 A.M.	
11.30 A.M.	9.10 A.M.	454	Lv. Kansas City Ar.	10.20 P.M.	8.45 P.M.
	2.25 P.M.	654	Lv. Newton (C.T.) Ar.		2.55 A.M.
5.00 A.M.	10.45 P.M.	1024	Lv. (M.T.) La Junta Ar.	6.30 A.M.	4.10 P.M.
8.10 P.M.	11.25 A.M.	1372	Lv. Albuquerque Ar.	6.00 P.M.	4.05 A.M.
11.10 A.M.	10.23 P.M.	1753	Ar. Williams Lv.	5.30 A.M.	4.35 A.M.
	1.30 A.M.	1817	Ar. Grand Cañon Lv.	7.50 P.M.	
12.15 P.M.	11.28 P.M.	1776	Ar. Ash Fork Lv.	4.05 A.M.	3.15 A.M.
	3.22 A.M.	1833	Ar. Prescott Lv.	12.23 A.M.	
	9.00 A.M.	1970	Ar. Phœnix (M.T.) Lv.	6.30 P.M.	
2.05 A.M.		2102	Ar. (P.T.) Barstow Lv.		2.20 P.M.
5.35 A.M.	12.10 P.M.	2203	Ar. San Bernardino Lv.	12.05 P.M.	11.00 P.M.
8.30 A.M.	2.35 P.M.	2263	Ar. Los Angeles Lv.	10.00 A.M.	9.00 A.M.
	3.00 P.M.	—	Lv. Los Angeles Ar.	7.15 A.M.	
1.10 P.M.	6.30 P.M.	2389	Ar. San Diego Lv.	2.45 A.M.	2.45 A.M.
2.15 A.M.		2102	Lv. Barstow Ar.		1.55 P.M.
4.25 A.M.	11.00 A.M.	2193	Ar. Mojave Lv.	11.45 A.M.	
10.40 A.M.	4.38 P.M.	2373	Ar. Fresno Lv.	4.15 A.M.	
2.25 P.M.	7.35 P.M.	2496	Ar. Stockton Lv.	12.45 A.M.	
5.30 P.M.	10.30 P.M.	2574	Ar. San Francisco Lv.	9.30 P.M.	8.00 P.M.

Reading the Table

In the time-table on page 209 the names of the stations are placed in the center, the time from Chicago, etc., to San Francisco being given on the left and that from San Francisco to Chicago on the right.

St. Louis is enclosed between lines to indicate that it is not on the main line. A passenger from St. Louis meets train 3 at Kansas City. Returning from the west, he leaves train 4 at the same point.

The stations in the next space are on the main line. A passenger for Grand Cañon leaves the train at Williams.

Ash Fork is on the main line. Passengers leave the train here to take one for Prescott and Phœnix. Barstow is on the main line; from it a train goes to San Bernardino, Los Angeles, and San Diego. Through passengers continue through Mojave, Fresno, etc., to San Francisco.

Sight Problems

1. What is the length of the branch line (*a*) from Williams to Grand Cañon? (*b*) From Ash Fork to Phœnix? (*c*) From Barstow to San Diego?

2. How long does (*a*) train 1 remain in Kansas City? (*b*) Train 2? (*c*) Train 3? (*d*) Train 4?

3. What is the distance (*a*) from Kansas City to Newton? (*b*) From Barstow to San Francisco?

4. How long does it take (*a*) train 1 to go from Chicago to Kansas City? (*b*) Train 3?

How long does it take (*c*) train 2 to go from Kansas City to Chicago? (*d*) Train 4?

5. When it is 10 : 30 P.M. at San Francisco what is the time at Chicago?

Written Problems

1. If a person leaves Chicago by train 1 on Monday, on what day does he reach San Francisco?

2. (*a*) How many hours does it require train 2 to go from San Francisco to Chicago? (*b*) How many miles does the train average per hour, including stops?

3. Find the expense of a trip from San Francisco to Chicago on train 4, allowing $59.75 for fare, $10.40 for sleeping accommodations, and three meals daily at the rate of 60 cents for breakfast, 75 cents for dinner, and 40 cents for supper.

In Italy, Spain, and Belgium the hours are numbered from 1 to 24, the latter being midnight. Some Canadian railroads follow this practice in their time-tables.

Travelers' Checks. — To render it unnecessary for a tourist, commercial traveler, etc., to carry a considerable sum in cash to meet hotel bills and other necessary expenses, associations of bankers issue books containing checks in denominations of $10, $20, $50, etc. Each check is signed by the buyer at the time he obtains it. When he presents one in payment of a hotel bill, etc., he signs it again in the presence of the person to whom he gives it.

Foreign Travel. — Bankers, express companies, etc., issue similar checks for use abroad, the equivalent in the money of various foreign countries being printed on the check. The following is a portion of the equivalents given on one for $20:

In U. S. and Canada	England			France		Germany		Russia	
	£	*s.*	*d.*	Fr.	Cent.	Marks	Pf.	Rubles	Kopecks
Twenty Dollars	4	1	8	102	50	83	30	38	46

Written Problems

1. Find the value of each of the following in U.S. money:

a. 102.50 francs at 19.3¢ *b.* 83.30 marks at 23.8¢
c. 38.46 rubles at 51.5¢ *d.* £4 1*s.* 8*d.* at $4.8665

2. What per cent of $20 is deducted in each case?

Letters of Credit. — Harold Cox, intending to spend a few months in Europe, obtains from his home bank a check payable in New York. This he presents to a New York banker, obtaining therefor a *letter of credit* for £250, travelers' checks for $200, and the remainder in the currency of the country of his steamer's destination. He selects English money for the letter of credit as the one for which he can get the best rates of exchange in the other countries.

With the letter of credit is given a list of bankers in Europe who will pay all or any portion of the sum named, in the money of their respective countries, at the current rate of exchange. Each payment is noted on the letter, to inform other bankers of the amount still remaining to the owner's credit.

When Mr. Cox returns home, he deposits in his own bank any unused travelers' checks and the letter of credit, if any balance remains, to be collected and placed to his account.

Written Exercise

How much does he pay the banker, at the rate of $4.86½ per £ with 1% for commission, $70.59 for the Italian money, and the face value of the travelers' checks plus a commission of ½%?

Solar Time

The connection between longitude and time is of importance chiefly to the mariner at sea. A chronometer on the vessel gives the London time. The captain, at noon by the sun, notes the difference between London time and that of the vessel, and calculates therefrom the longitude of the vessel.

Sight Exercises

1. Give the difference in degrees of longitude between a vessel and London when the time difference is

a. 1 hr. *b.* 1½ hr. *c.* 2 hr. *d.* 6 hr.

2. Give the difference in longitude when it is noon on a vessel and the chronometer gives Greenwich (London) time as follows:

a. 1 P.M. *b.* 11 A.M. *c.* 2:30 P.M.
d. 9:30 A.M. *e.* 4:30 A.M. *f.* 7:30 P.M.

3. When a vessel is west of the meridian of Greenwich (*a*) is it noon on a vessel before or after it is noon at Greenwich? (*b*) Does the chronometer show time before or after 12 o'clock noon?

4. Give the longitude of a vessel at noon, when the chronometer gives London time as follows:

a. 2 P.M. *b.* 3 P.M. *c.* 4 P.M. *d.* 5 P.M.
e. 11 A.M. *f.* 9 A.M. *g.* 6 A.M. *h.* 5 A.M.

5. How many minutes of time correspond to a longitude difference (*a*) of 15°? (*b*) Of 1°? (*c*) Of 2°? (*d*) Of ½°? (*e*) Of 6°?

Transporting Merchandise

The coöperation of railroads, which enables a passenger to ride for nearly a week without changing cars is extended to the transportation of freight.

An eastern farmer desiring to settle in Oklahoma, Arizona, etc., can load up all his possessions, including live stock, in one or more freight cars, traveling with them to his destination.

Small articles ordered from a distant city are delivered at the door by the parcel post or by the express; larger ones are brought to the nearest railroad station.

Grain is hauled from the farm to the railroad, the loaded cars deliver it to the lake vessel, thence it goes to the canal boat, to be placed upon the ocean steamer for delivery at a European port. In many cases the chief expense is the cost of getting it from the farms to the railroad station.

Parcel Post

Weight	First Zone Local Rate	Weight	Second Zone 150 mi.	Third Zone 150 to 300 mi.	Fourth Zone 300 to 600 mi.	Fifth Zone 600 to 1000 mi.	Sixth Zone 1000 to 1400 mi.	Seventh Zone 1400 to 1800 mi.	Eighth Zone over 1800 mi.
1 lb.	$0.05	1 lb.	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 & 3 "	.06	2 "	.06	.08	.11	.14	.17	.21	.24
4 & 5 "	.07	3 "	.07	.10	.15	.20	.25	.31	.36
6 & 7 "	.08	4 "	.08	.12	.19	.26	.33	.41	.48
8 & 9 "	.09	5 "	.09	.14	.23	.32	.41	.51	.60
10 & 11 "	.10	6 "	.10	.16	.27	.38	.49	.61	.72
12 & 13 "	.11	7 "	.11	.18	.31	.44	.57	.71	.84
14 & 15 "	.12	8 "	.12	.20	.35	.50	.65	.81	.96
16 & 17 "	.13	9 "	.13	.22	.39	.56	.73	.91	1.08
18 & 19 "	.14	10 "	.14	.24	.43	.62	.81	1.01	1.20
etc.									

Sight Problems

1. How many feet long may a package be so that the sum of its length and the perimeter of the end may not exceed 6 feet?

A parcel must not exceed 72 inches in combined length and girth.

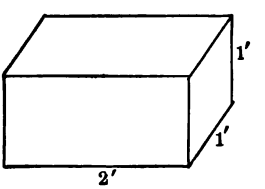
2. How long may a package be to keep within the limit of 6 feet when each end is a square measuring $\frac{1}{2}$ foot?

3. Give the volume of a box whose length is 2 feet, the end being 1 foot square.

4. Give the volume of a box whose length is 4 feet, each end being $\frac{1}{2}$ foot square.

Written Problems

1. A dealer in artificial flowers wishes to ascertain the dimensions of the ends of a box 24 inches long that will come under the provisions of the parcel post as to size, and will contain the greatest quantity.



Find the capacity of each of the following boxes in cubic feet and a fraction:

a. 24 inches long, ends 9 in. by 15 in.
b. 24 inches long, ends 8 in. by 16 in.
c. 24 inches long, ends 6 in. by 18 in.
d. 24 inches long, ends 12 in. by 12 in.

NOTE. — Change each dimension to feet before multiplying.

2. (*a*) What part of a cubic foot will be contained in a box 11 inches long, 9 inches wide, 4 inches deep? (*b*) At $\frac{4}{5}$ bushel to the cubic foot, what fraction of a bushel will it hold? (*c*) What is the weight of the wheat it will contain at 60 pounds to the bushel?

Express

Express companies supply transportation of articles too heavy or too bulky to be sent by parcel post, also of goods requiring prompt delivery. A company's wagon collects packages at the residences or stores of the senders and delivers them at the proper railroad station. Another wagon in the distant city takes each package to the designated address.

Freight

Bulky articles and goods in large quantities are transported by freight. The shipper delivers them at the railroad station or steamer dock, and receives from the agent of the company a receipt called a *Bill of Lading*. This is mailed to the person to whom the goods are to be delivered.

Written Problems

1. Change 3*d*., English money, to cents and a decimal.

2. Find the cost per bushel of sending grain from Chicago to Liverpool at \$.0112 for the lake freight, \$.043 for the canal freight, 3*d*. for the ocean freight, $\frac{1}{2}¢$ for handling charges at Buffalo and 1¢ in New York.

3. When the all-rail rate from Chicago to New York is 9.6 cents per bushel for wheat weighing 60 pounds to the bushel, what is the freight per ton?

4. What is the cost per bushel per mile for transporting wheat from Chicago to New York, 960 miles?

5. At the rate of 9.6 cents for 960 miles, what would be the rate on a bushel of wheat to New York from a point 1776 miles west of Chicago?

6. Find the canal freight on (*a*) 8500 bushels of corn at 3.8¢ per bushel. (*b*) 8000 bushels of wheat at 4.3¢ per bushel.

Transmitting Money

If Thomas Blake in Bismarck desires to send $50 to Daniel Lawler in Chicago, he may do so in any one of the following ways:

He can send his personal check drawn on a Bismarck bank, which Mr. Lawler collects in Chicago, at a small discount.

He can buy from a Bismarck bank a cashier's check payable at a Chicago bank; this may cost him 25 cents in addition to the $50.

He can buy for $50.18 a postal money order payable at the Chicago post office, or an express money order payable at the express company's Chicago office.

He can transmit the money by telegraph by paying $50 at the telegraph office together with the regular fee and the cost of two telegrams. He notifies Mr. Lawler by telegraph, and the latter obtains the money at a telegraph office in Chicago.

Bank Drafts

One form of paying a large sum is by means of a bank draft. The following is a draft purchased from the First National Bank of Seattle by Fred Johnson, to pay Mr. Priedigkeit of Xenia the sum specified.

SEATTLE, WASH., *Aug. 15*, 19*15*

1248 $\frac{75}{100}$

At sight pay to the order of *William Priedigkeit*

Twelve Hundred Forty-eight $\frac{75}{100}$ Dollars.

Value received, and charge to the account of the *First National Bank.*

TO MERCHANTS' BANK
XENIA, OHIO

Martin E. Lyneh
Cashier

The cost of the foregoing draft depends upon the *rate of exchange*, which varies from time to time. When a draft is purchased for less than its face, exchange is at a *discount;* when it costs more than its face, exchange is at a *premium.* The rates quoted in the daily papers give the premium or the discount on $1000, unless a rate per cent is specified.

The draft shown on page 111 is called a *sight draft*, which is payable upon presentation.

Written Exercises

1. Find the cost of the foregoing draft at a premium of 75¢ per thousand dollars.

$1248.75 + ¼ of $1.25

2. (*a*) What is the cost of a draft for $20,000 purchased at a discount of ⅛%? (*b*) What rate per thousand corresponds to ⅛%?

Cashier's Checks

The sight draft is now being displaced by a cashier's check, as follows:

FIRST NATIONAL BANK
SEATTLE, WASH., *Aug. 15,* 19*15*

MERCHANTS' BANK
XENIA, O.

Pay to the order of *William Riedigkeit*

Twelve Hundred Forty-eight $\frac{75}{100}$ ~~~~~~~~~~ Dollars.

$*1248*\frac{75}{100}$

Wm. J. Brown
Cashier

Bills of Exchange

A draft payable in a foreign country is called a *bill of exchange.* This is issued in a set of two, the *original* and the *duplicate.* When one is paid, the other has no value.

San Francisco, Cal., *Sept. 4,* 19*11*

£534 3s. 6d.

At Sight of this first of exchange, second unpaid, pay to the order of *Maurice F. Egan*

Five Hundred Thirty-four Pounds Sterling, three shillings, sixpence,

and charge the same to the account of

To *Pye, Neal, & Co.*
London, Eng.

Jos. F. Lamorelle

Bills of exchange are generally payable at sight or at a given time after sight. The following is one form of a time bill of exchange:

Exchange for 5000 fr.

Albuquerque, N. M., *Aug. 4,* 19*16*

Sixty days after sight of this second of exchange, first unpaid, pay to the order of *Charles Mette*

Five Thousand Francs,

and charge to the account of

To *Franeolini Frères*
Marseilles, France

Barniele & Johnson

When Mr. Mette receives this bill, he presents it to Francolini Frères for *acceptance.* They agree to pay the bill when due by writing across the face in red ink the word "Accepted," with the date, say, "Aug. 11, 1916." Sixty days after Aug. 11, the bill is payable upon presentation.

The prices of exchange are given in the papers in the following form, the rates varying from time to time :

Rates of Exchange

	Sight	60 Days
Sterling	4.86	4.84
Francs	5.17½	5.20
Marks	.95	.94½

These figures mean that a sight bill on a place in Great Britain costs $4.86 per £, and a 60-day bill $4.84.

The rate for French money means that $1 will purchase a sight bill for 5.17½ francs, or a 60-day bill for 5.20 francs.

The rate for German money means that 95 cents is the cost of 4 marks in a sight bill and 94½ cents in a 60-day bill.

Written Exercises

1. Find the cost of a sight bill for £534 7*s.* 6*d.* at $4.86 per £.

2. How much is paid for a 60-day bill on Marseilles for 5000 fr. at the rate of 5 fr. 20 for $1?

$(5000 \div 5.2)$

3. A bill for 1876 marks is bought at the rate of 4 marks for 94½ cents. How much did it cost?

4. How many francs will cost $2500 at the rate of 5.17½ francs to the dollar?

Use of Drafts in Collecting Money

To insure prompt payment, the Wainscott Manufacturing Co. "draws" upon its customers as their bills fall due.

The bill of Henry Jenkins of Hightown, N. D., for $448.75 is due May 6, 1915. The Wainscott Manufacturing Co. makes out the following draft:

MINNEAPOLIS, MINN., *May 1*, 19*15*

At *three days'* sight pay to the order of

THE MANUFACTURERS' BANK

Four Hundred Forty-eight $\frac{75}{100}$ Dollars

and charge to the credit of

To *Henry Jenkins*
Hightown, N.D.

WAINSCOTT MANUFACTURING CO.
J. F. Rorke, Treas.

This draft is deposited "for collection" in the Manufacturers' Bank, in which the Wainscott Manufacturing Company keeps its account. This bank sends it to the First National Bank of Hightown, which it reaches May 3. A messenger from this bank presents it for acceptance to Henry Jenkins, who writes across its face "Accepted, May 3, 1915," and appends his signature. If he keeps an account in the First National Bank, the latter "charges" it with $448.75, and mails the Manufacturers' Bank a cashier's check for the amount.

By this method the Wainscott Manufacturing Company obtains the money by May 8, since Mr. Jenkins would not like to injure his credit with his home bank by refusing to "accept" the draft.

Accurate Interest

While the convenience attending the use of the year of 360 days has led to its employment in ordinary business calculations of interest on small sums for portions of a year, the 365-day year is the basis in determining the interest payments of the United States and of banks, although the latter generally take the year of 360 days in discounting notes.

Written Exercises

1. The Nassau Trust Company owes Harrison Javins $240, with interest from July 16, 1914. What is the amount of its indebtedness Feb. 20, 1915?

Time, 219 days. $\quad Interest = \dfrac{\$\,240 \times .03 \times 219}{365}$.

2. Find the interest on $240 for 219 days, at 3%, taking 360 days to the year.

3. Find the accurate interest on

a. $1460, from Jan. 3, 1915 to Aug. 5, 1915, at 6%.

b. $2000, from March 8, 1914 to May 20, 1914, at 5%.

c. $3650, from May 5, 1916 to Oct. 2, 1916, at 2%.

4. From the accompanying table find the accurate interest on $2760 for 21 days at 5%.

NOTE. — Take from the table the interest on $1600 ($4.603), on $1000 ($2.887), and on $160 ($.460). Combine the three.

5. Find from the table the interest on $1200 for 28 days at 5%.

365 DAYS TO YEAR — 21 DAYS

	5%	6%	7%
$1000	2.887	3.452	4.027
1100	3.164	3.797	4.430
1200	3.452	4.142	4.833
1300	3.74	4.488	5.236
1400	4.027	4.833	5.638
1500	4.315	5.178	6.041
1600	4.603	5.523	6.444

PROCESS

Interest for 21 da. at 5 % on $1200 is $3.452.
Interest for 7 da. is $\frac{1}{3}$ of foregoing. Combine.

6. Find the accurate interest on

a. $400 for 21 da. at 5 %
b. $350 for 21 da. at 6 %
c. $600 for 7 da. at 3 %
d. $800 for 21 da. at $3\frac{1}{2}$ %
e. $700 for 42 da. at 6 %
f. $900 for 28 da. at $2\frac{1}{2}$ %

Table of Interest for 1 Day

365 Days to Year
Interest for 1 Day at 2 %

Principal	Interest	Principal	Interest
$10,000	.55	$210,000	$11.51
11,000	.60	211,000	11.56
12,000	.66	212,000	11.62
13,000	.71	213,000	11.67
14,000	.77	214,000	11.73
15,000	.82	215,000	11.78
16,000	.88	216,000	11.84
17,000	.93	217,000	11.89
18,000	.99	218,000	11.95
19,000	1.04	219,000	12.00

7. Mr. Haupt's bank allows him interest at 2 % on a deposit of $25,000. How much interest is due him per day?

PROCESS

Int. on $10,000	.55
Int. on 15,000	.82
Int. on $25,000	$1.37

8. What is the accurate interest on $2500 for 90 days at 2 %?

The interest on $2500 for 90 days is the same as the interest on $225,000 (90 times $2500) for 1 day.

9. Find the accurate interest at 2 % on

a. $3000 for 75 days
b. $6000 for 38 days
c. $4000 for 56 days
d. $7000 for 33 days

Interest-bearing Deposits

A commercial bank does not, as a rule, pay interest on a depositor's small balances subject to withdrawal at any time by check. It will, however, pay a low rate of interest on a special deposit left with it for a definite period.

James H. Tully has $3000, for which he has no immediate use. This he places in the Citizens Bank, receiving the following certificate:

CERTIFICATE OF DEPOSIT

No. 8376 BILLINGS, OKLA., March 1, 1915

This certifies that James H. Tully has deposited in this Bank Three Thousand and $\frac{00}{100}$ Dollars payable sixty days after date to his order upon return of this certificate, with interest at 2 per cent per annum.

CITIZENS BANK
E. R. Smith,
Cashier.

Written Exercises

1. Find the amount of the foregoing certificate of deposit at the time of its withdrawal, May 24, 1915.

Time between March 1 and May 24 is 84 da.

$3000 × 84 = $252,000.

Take from the table the interest for 1 day on $210,000 and to this add the interest on $42,000 ($\frac{1}{5}$ of $210,000).

2. Find the amount of each of the following certificates of deposit at the expiration of its term:

a. Face $4000, term 63 days, rate 2%.

b. Face $3500, term 60 days, rate 3%.

c. Face $7000, term 30 days, rate $2\frac{1}{2}$%.

Interest on Bank Balances

In the case of depositors whose balances exceed a certain amount (say $500), some commercial banks pay interest at the rate of 2% per year on the balances.

Written Exercises

1. William S. Hurley's account with the Home Trust Company shows the following transactions:

Jan. 1, $1200 balance.	Apr. 5, Deposited $300.
Feb. 2, Deposited $300.	May 6, Withdrew $400.
March 4, Withdrew $600.	June 7, Deposited $800.

If the bank allows 2% on balances, the interest to be credited to his account on July 1 is determined as follows:

PROCESS

Dates	Time	Balances	Int. for 1 Da. on
Jan. 1 to Feb. 2	32 da.	$1200	$38,400
Feb. 2 to Mch. 4	30 da.	1500	45,000
Mch. 4 to Apr. 5	32 da.	900	28,800
Apr. 6 to May 6	31 da.	1200	37,200
May 6 to June 7	32 da.	800	25,600
June 7 to July 1	24 da.	1600	38,400
		Interest for 1 day on $213,400 =	$11.69

The balance of $1200 on Jan. 1 is entitled to interest to Feb. 2, 32 days; the new balance of $1500 is entitled to interest to March 4, 30 days; etc., as shown above.

Instead, however, of finding the interest on each balance, it is changed with its time into the sum entitled to 1 day's interest, and the six sums are combined into one, the interest on which is determined by means of the table.

2. David Roche's account shows balances as follows: July 1, 1914, $1000; Aug. 2, $2000; Sept. 5, $1200; Oct. 12, $1000; Nov. 15, $800; Dec. 3, $1000. How much interest at 2% is credited on these balances Jan. 1, 1915?

Interest Laws

When the rate is not expressed in the paper showing the indebtedness, the *legal rate* of the state is employed.

In some states, a contract to pay a higher rate is not enforceable; in others, the law authorizes the collection of a rate not exceeding a specified one; in a few, any rate agreed upon by the borrower is binding upon him.

The collection of excessive interest is called *usury*.

LEGAL AND CONTRACT RATES

STATES	LEGAL RATE	CONTRACT RATE	STATES	LEGAL RATE	CONTRACT RATE
Alabama	8	8	Nebraska	7	10
Arkansas	6	10	Nevada	7	Any
Arizona	6	Any	New Hampshire	6	6
California	7	Any	New Jersey	6	6
Colorado	8	Any	New Mexico	6	12
Connecticut	6	6	New York	6	6
Delaware	6	6	North Carolina	6	6
Dis. of Col.	6	10	North Dakota	7	12
Florida	8	10	Ohio	6	8
Georgia	7	8	Oklahoma	7	12
Idaho	7	12	Oregon	6	10
Illinois	5	7	Pennsylvania	6	6
Indiana	6	8	Rhode Island	6	Any
Iowa	6	8	South Carolina	7	8
Kansas	6	10	South Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	Any	Utah	8	12
Maryland	6	6	Vermont	6	6
Massachusetts	6	Any	Virginia	6	6
Michigan	5	7	Washington	6	12
Minnesota	7	10	West Virginia	6	6
Mississippi	6	10	Wisconsin	6	10
Missouri	6	8	Wyoming	8	12
Montana	8	Any			

Annual Interest

Written Exercises

1. DETROIT, MICH., Jan. 15, 1915

On demand after date I promise to pay to the order of Andrew J. Shipman, Four Hundred Dollars, value received, with interest at six per cent, payable semiannually.

$\$400_{\frac{00}{100}}$ CLEMENT MANLY

Find the amount due on the foregoing note on Jan. 15, 1918, if no interest payments have been made.

In most states Mr. Shipman could collect only $72 interest, the laws not permitting the collection of interest on overdue interest. In Michigan, however, and in some other states, the law permits the collection of simple interest on deferred interest payments from the time they are due until they are paid, when the note expressly provides that the interest is to be paid at stated times.

PROCESS

The five semiannual payments of $12 being unpaid for $2\frac{1}{2}$ yr., 2 yr., $1\frac{1}{2}$ yr., 1 yr., and $\frac{1}{2}$ yr., respectively, interest on $12 at 6% is due for $2\frac{1}{2}$ yr. + 2 yr. + $1\frac{1}{2}$ yr. + 1 yr. + $\frac{1}{2}$ yr., or for $7\frac{1}{2}$ yr., together with the interest on $400 for 3 yr.

Principal	$400.00
Interest on $400, for 3 yr. at 6%	72.00
Interest on $12 for $7\frac{1}{2}$ yr. at 6%	5.40
Amount due Jan. 15, 1912	$477.40 *Ans.*

2. How much is due Aug. 4, 1916, on a note drawn Aug. 4, 1912, for $600 with 6% interest payable annually, if no payments of interest have been made, and the collection of annual interest is permitted by law?

3. A note for $900 was made Sept. 16, 1913, with interest at 6% payable semiannually. How much was due Sept. 16, 1916, if no interest payments were made except the first?

Compound Interest Tables

Amount of $1 at compound interest, compounded annually.

Years	Rate: 2 %	2½ %	3 %	4 %	5 %
1	1.02000	1.02500	1.03000	1.04000	1.05000
2	1.04040	1.05063	1.06090	1.08160	1.10250
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15969	1.21665	1.27628
6	1.12616	1.15969	1.19405	1.26532	1.34010
7	1.14869	1.18869	1.22987	1.31593	1.40710
8	1.17166	1.21840	1.26677	1.36857	1.47746
9	1.19509	1.24886	1.30477	1.42331	1.55132
10	1.21899	1.28008	1.34392	1.48024	1.62889
11	1.24337	1.32125	1.38423	1.53945	1.71033

Written Exercises

1. What is the interest on $800 compounded annually for 5 years at 5%?

$$\text{Interest} = \$800 \times .27628.$$

2. Find the interest on the following, compounded annually:

a. $800 at 3% for 4 yr. *b*. $800 at 2% for 6 yr.
c. $800 at 4% for 3 yr. *d*. $800 at 5% for 7 yr.

3. What is the amount of $800 compounded semi-annually for 5 years at 5%?

$$\text{Amount} = \$800 \times 1.28008.$$

Take from the table the amount of $1 for 10 years at $2\frac{1}{2}$%, since $800 is compounded 10 times at $2\frac{1}{2}$%.

Miscellaneous Applications

While the pupil should not burden his memory with the units of tables infrequently used, he has not obtained the greatest possible benefit from his study of arithmetic if he is unable to work an example involving any denominate units whatever, in case he has the table before him.

In the same way, he should be expected to solve a problem involving easily understood conditions, provided they are clearly stated in the problem.

The following examples are given as a test of the pupil's ability to read understandingly.

Written Problems

1. At the rate of 1800 cubic feet per person per hour, how much air will be required in a bedroom by 2 people occupying it for 8 hours?

2. (*a*) How many cubic feet of air are in a bedroom 10 ft. × 12 ft. × 9 ft.? (*b*) How many times must the air be changed to supply 2 persons with 1800 cu. ft. each per hour? (*c*) If the windows are opened sufficiently to change the air 4 times per hour, how many cubic feet are supplied for each?

3. How many persons will receive 1800 cu. ft. of fresh air per hour in a room $24' \times 20' \times 12'$, when the open windows change the air four times per hour?

4. (*a*) How many cubic feet of air per hour are required by 47 pupils and a teacher at the rate of 1800 cubic feet per hour? (*b*) How many cubic feet are there in a classroom $24' \times 20' \times 12'$? (*c*) How many times per hour must the air in the classroom be changed to supply the necessary quantity of pure air?

5. (*a*) How many cubic feet of air per second are required to furnish 1800 cu. ft. per person per hour to 30 persons? (*b*) If the ventilators admit air at the rate of 5 cubic feet per second for each square foot of opening, how many square feet of the latter are necessary to supply the air? (*c*) How many square feet must the ventilator measure if only three fourths of its surface is unobstructed?

6. Measure the dimensions of your classroom. (*a*) How many square feet of floor space are provided for each pupil and the teacher? (*b*) How many cubic feet of air space are provided for each?

7. (*a*) How much time does a man save in a year for other things, if he requires 30 minutes less per day to do the work in the barn after he has made some changes in its arrangements? (*b*) What is the extra time worth at 20 cents per hour? By improving the house conditions he has made it possible for his wife to save 40 minutes per day. (*c*) How many hours does she save in a year?

8. (*a*) What is the yearly interest on \$100 at 6%? (*b*) How much more does a man receive than the interest if an expenditure of \$100 for improved machinery will enable him to do 180 hours of extra work worth 20 cents per hour?

9. What interest does a man receive on his yearly payment of \$25 for improving the roads, if he saves the equivalent of a man and a team for 34 days, worth \$3 per day, in hauling his produce to market, etc.?

10. A foot pound is the work done in lifting 1 foot 1 pound. How many foot pounds of work are done by a man who carries 6000 pounds of bricks and mortar to the top of a 22-foot ladder?

11. A horse power is the power required to do 33,000 foot pounds in 1 minute. What fraction of a horse power is exerted in lifting 6000 pounds 22 feet in 8 hours?

12. The horse power of a gasoline engine may be calculated from the following formula:

$$\text{H.P.} = \frac{D^2 \times N}{2.5}$$

in which D represents the diameter of the cylinder in inches and N the number of cylinders.

Find the horse power of a 6-cylinder automobile engine when the diameter of each cylinder is 5 inches.

13. (*a*) Taking the weight of a gallon of water as $8\frac{1}{4}$ lb., find the weight of the water to be pumped per minute for 10 hours to supply 6000 gallons. (*b*) How many foot pounds of labor are required per minute to raise it 40 ft.? (*c*) What decimal of a horse power?

14. (*a*) How many feet per minute does a horse travel when he walks $2\frac{1}{2}$ miles in an hour? (1 mile = 5280 ft.) (*b*) How many foot pounds of power does a horse exert in a minute in carrying a load of 150 pounds 220 feet?

15. The load the average horse can carry on his back for 10 hours per day is about $\frac{1}{10}$ of his weight. This is the power he exerts in drawing a wagon containing a ton on a smooth pavement, or one half ton on an ordinary road. What horse power will a 1500-pound horse develop in overcoming a resistance of 150 pounds, when he travels $2\frac{1}{2}$ miles per hour for 10 hours?

16. Marching along a good road soldiers take 132 steps per minute, each step being 2 ft. 6 in. (*a*) How many feet do they march in a minute? (*b*) How many minutes would they require to march a mile?

Interest by Six Per Cent Method

Preparatory Exercises

Six per cent per year is what per cent for (*a*) 2 yr.? (*b*) 3 yr.? (*c*) 4 yr.?

Six per cent per year is what per cent for (*a*) 1 mo.? (*b*) 2 mo.? (*c*) 4 mo.? (*d*) 6 mo.? (*e*) 8 mo.?

Six per cent per year (*a*) is 1% for how many days? (*b*) Is what per cent for 120 days? (*c*) Is what per cent for 6 days? (*d*) Is what per cent for 24 days?

By the *six per cent method* the interest at 6% is found on \$1 for the given time, and this is multiplied by the number of dollars in the given principal.

Written Exercises

1. Find the interest on \$396.50 at 6% for 3 yr. 10 mo. 23 da.

PROCESS

Int. on \$1 at 6% for 3 yr.	\$0.18	
Int. on 1 at 6% for 10 mo.	.05	$\frac{1}{2}$ of 10¢
Int. on 1 at 6% for 23 da.	.003$\frac{5}{6}$	$\frac{1}{6}$ of 2.3¢
Int. on \$1 for 3 yr. 10 mo. 23 da.	\$0.233$\frac{5}{6}$	

\$0.233$\frac{5}{6}$ × 396.5 = \$88.85.

At 6¢ per year, the interest for 3 yr. is 18¢.
At $\frac{1}{2}$¢ per month, the interest for 10 mo. is 5¢.
At $\frac{1}{6}$ of 1 mill per day, the interest for 23 da. is 3$\frac{5}{6}$ mills.

2. Find the interest at 6%,

a. On \$275, for 1 yr. 4 mo. 18 da.
b. On \$346, for 2 yr. 6 mo. 24 da.
c. On \$409, for 3 yr. 2 mo. 12 da.
d. On \$542, for 4 yr. 1 mo. 6 da.

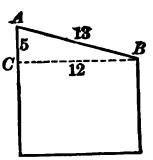
SECTION VI

PRACTICAL USES OF POWERS AND ROOTS

Squares and Square Roots

Preparatory Exercises

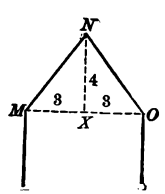
1. The front of a shed is 5 feet higher than the back. The shed is 12 feet deep. How long is the roof?



Draw ABC on a scale of 4 inches to the foot. Make CB 3 inches long. At C erect a perpendicular CA, $1\frac{1}{4}$ inches. Measure AB.

2. MNO is the upper part of the front of a tent. MO is 6 feet. NX measures 4 feet. How long is NO?

Draw a line XO, 3 inches long. At X erect a perpendicular XN, 4 inches long. Draw the line NO, and measure its length.



The triangles ACB and NXO are *right triangles*. The angles at C and X are *right angles*. The side AB or NO, opposite the right angle, is the *hypotenuse*. One of the remaining sides of a right triangle is the *base*, and the other the *perpendicular*.

In the right triangle ABC, it will be found that $(13 \times 13) = (12 \times 12) + (5 \times 5)$; that is, $169 = 144 + 25$.

13×13 is written 13^2: it is read 13 "*square*." 5×5 is written 5^2; 12×12 is written 12^2.

Hypotenuse^2 = Base^2 + Perpendicular^2

Sight Problems

1. How many square rods are there in a plot of ground 8 rods long 8 rods wide?

2. A piece of ground 12 rods long 12 rods wide is said to be 12 rods square. How many square rods are there in a piece of ground 12 rods square?

3. How many square inches are there in each of the six equal square faces of a cubical box whose edge measures 3 inches?

4. Give answers:

a. $3^2 = ?$	*b.* $6^2 = ?$	*c.* $10^2 = ?$	*d.* $20^2 = ?$	*e.* $90^2 = ?$
f. $4^2 = ?$	*g.* $7^2 = ?$	*h.* $11^2 = ?$	*i.* $30^2 = ?$	*j.* $80^2 = ?$
k. $5^2 = ?$	*l.* $9^2 = ?$	*m.* $12^2 = ?$	*n.* $40^2 = ?$	*o.* $70^2 = ?$

5. How many square rods are there in a 10-acre field?

6. How many rods are there in each side of a square field (*a*) containing 1600 sq. rd.? (*b*) Containing 10 A.?

7. How many inches long is each side of a square piece of paper that contains (*a*) 16 sq. in.? (*b*) 36 sq. in.? (*c*) 100 sq. in.?

8. What are the two equal factors (*a*) of 49? (*b*) Of 25? (*c*) Of 81? (*d*) Of 121?

9. Give answers:

a. $81 = (?)^2$	*b.* $100 = (?)^2$	*c.* $1600 = (?)^2$
d. $49 = (?)^2$	*e.* $121 = (?)^2$	*f.* $8100 = (?)^2$
g. $36 = (?)^2$	*h.* $144 = (?)^2$	*i.* $6400 = (?)^2$

Square Root

To indicate that one of the two equal factors of a number is required, the sign $\sqrt{\ }$ is used.

Thus, $\sqrt{81} = 9$ is read "The *square root* of 81 is 9."

Sight Exercises

Give answers:

a. $\sqrt{4} = ?$ *b.* $\sqrt{36} = ?$ *c.* $\sqrt{144} = ?$ *d.* $\sqrt{2500} = ?$

e. $\sqrt{9} = ?$ *f.* $\sqrt{64} = ?$ *g.* $\sqrt{169} = ?$ *h.* $\sqrt{3600} = ?$

i. $\sqrt{1} = ?$ *j.* $\sqrt{49} = ?$ *k.* $\sqrt{121} = ?$ *l.* $\sqrt{8100} = ?$

Architects, surveyors, mechanics, and other business people use tables to ascertain the squares and the square roots of numbers. The following table gives the squares and the square roots of numbers to 60.

Squares and Square Roots

No.	Sq.	Sq. Root	No.	Sq.	Sq. Root	No.	Sq.	Sq. Root
1	1	1.	21	441	4.5826	41	1681	6.4031
2	4	1.4142	22	484	4.6904	42	1764	6.4807
3	9	1.7321	23	529	4.7958	43	1849	6.5574
4	16	2.	24	576	4.8990	44	1936	6.6332
5	25	2.2361	25	625	5.	45	2025	6.7082
6	36	2.4495	26	676	5.0990	46	2116	6.7823
7	49	2.6458	27	729	5.1962	47	2209	6.8557
8	64	2.8284	28	784	5.2915	48	2304	6.9282
9	81	3.	29	841	5.3852	49	2401	7.
10	100	3.1623	30	900	5.4772	50	2500	7.0711
11	121	3.3166	31	961	5.5678	51	2601	7.1414
12	144	3.4641	32	1024	5.6569	52	2704	7.2111
13	169	3.6056	33	1089	5.7446	53	2809	7.2801
14	196	3.7417	34	1156	5.8310	54	2916	7.3485
15	225	3.8730	35	1225	5.9161	55	3025	7.4162
16	256	4.	36	1296	6.	56	3136	7.4833
17	289	4.1231	37	1369	6.0828	57	3249	7.5498
18	324	4.2426	38	1444	6.1644	58	3364	7.6158
19	361	4.3589	39	1521	6.2450	59	3481	7.6811
20	400	4.4721	40	1600	6.3246	60	3600	7.7460

Sight Exercises

1. Give the square roots of the following numbers:

a. 2601 *b.* 1444 *c.* 3136 *d.* 961 *e.* 60 *f.* 38

2. Give the squares (approximately) of

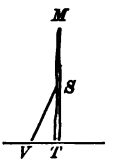
a. 3.3166 *b.* 5.5678 *c.* 7.4162 *d.* 6.9282 *e.* 7.4833

NOTE. — The product of 4.6904 by 4.6904 is 21.99985216, which is 22 for all practical purposes.

Applications of Square Root

Written Problems

1. How long is the wire rope SV, which is employed to keep the pole MT perpendicular? One end is fastened to the pole at S, 15 ft. from the ground. The other is fastened at V, which is 8 ft. from the base of the pole.



PROCESS

$B^2 = 64 \quad (8^2)$
$P^2 = 225 \quad (15^2)$
$H^2 = 289$

Add the squares of VT and TS. Look in the column of squares for 289. This is found to be the square of 17.

Ans. 17 ft.

2. Find the length of the hypotenuse of each of the following right triangles, the other sides of which measure, respectively:

a. 12 ft. and 16 ft. *b.* 7 mi. and 24 mi.
c. 35 rd. and 12 rd. *d.* 24 in. and 10 in.
e. 9 rd. and 40 yd. *f.* 20 in. and 21 in.

3. Find the base of a right triangle whose perpendicular measures 11 yards and hypotenuse 61 yards.

PROCESS

$B^2 = ?$
$P^2 = 121\ (11^2)$
$H^2 = 3721\ (61^2)$

Ans. 60 yd.

Since 3721 (H^2) represents the sum of 121 (P^2) + B^2, the latter equals 3600, the square root of which is 60.

4. Find answers:

a. Hypotenuse, 85; base, 84. Perpendicular?

b. Perpendicular, 112; hypotenuse, 113. Base?

Factors and Multiples

Preparatory Exercises

1. What is the square (*a*) of 2? (*b*) Of 5? (*c*) Of 10?

2. Show that $10^2 = 2^2 \times 5^2$.

3. What is the square root (*a*) of 100? (*b*) Of 25? (*c*) Of 4?

4. Show that $\sqrt{100} = \sqrt{4} \times \sqrt{25}$.

Written Problems

1. A piece of tin is 69 inches square. How many square inches does it contain?

PROCESS

$69^2 = 23^2 \times 3^2$
$69^2 = 529 \times 9$, etc.

From the table, $23^2 = 529$

2. Find the answers:

a. 336^2. *b.* 225^2. *c.* 153^2. *d.* 96^2. *e.* 184^2.

3. What is the length of a square field that contains 4761 square rods?

4. Find the square root of

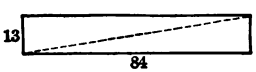
a. 36×169. *b.* 4×1369. *c.* 25×2809. *d.* 16×3481.
e. 25×289. *f.* 9×1681. *g.* 81×3249. *h.* 64×2209.

5. A number is divisible by 4 or by 25 when its last two figures are ciphers or constitute a multiple of 4 or of 25. A number is divisible by 9 when the sum of its digits is divisible by 9.

Using 4, 9, or 25 as a factor, find the square root of the following:

a. 7056. *b.* 54756. *c.* 60025. *d.* 86436.
e. 4761. *f.* 34596. *g.* 38025. *h.* 60516.

6. What is the length of the diagonal of a rectangular field whose dimensions are 84 rods and 13 rods, respectively?



PROCESS

The diagonal is the hypotenuse of a right triangle having sides of 84 rods and 13 rods, respectively.

$B^2 = 7056$
$P^2 = 169$
$H^2 = 7225$

Find the sum of the squares of the sides, 7225. Take 25 as one factor, which gives 289 as the other factor. Indicate the square root $\sqrt{25 \times 289}$, which equals 5×17. *Ans.* 85 rods.

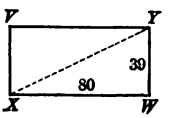
7. A 74-foot ladder just reaches a window 70 feet from the ground. How far from the wall is the foot of the ladder?

8. A vessel has sailed 96 miles due north from the starting point, and has then sailed 28 miles due east. How far is the vessel then from the starting point?

Extracting the Square Root

Written Problems

1. How much shorter is a path (XY) along the diagonal of a rectangular field 80 rods long and 39 rods wide than the distance $XW + WY$?



PROCESS

$80^2 = 6400$
$39^2 = 1521$
$H^2 = 7921$

I. To find the square root of 7921 begin at the ones and point off two figures, which leaves 79 as the first period.

II. As the largest square contained in 79 is 64, write 64 under 79, and place 8 (the square root of 64) above 79 (the first period).

Ans. 8 9

79'21
64
169 1521
1521

III. Subtract 64 from 79 and bring down the second period, making 1521 the partial dividend.

IV. Write 16 (twice 8) as a trial divisor. Divide 152 by 16 for the next figure of the root. Place 9 (the quotient) over the second period, and annex it to the trial divisor, making 169 the complete divisor.

V. Multiply 169 by 9. Since the product equals the partial dividend, the required root is 89. $XY = 89$ rods.

2. A bird is at the top of a tree 75 feet high. How far is it from a boy who is 40 feet from the foot of the tree? (Make a diagram.)

3. The foot of a ladder is 13 feet from the bottom of a building 84 feet high and just reaches the roof. (*a*) How long is the ladder? (*b*) How far from the bottom of a building is one end of a 50-foot ladder that just reaches a window 48 feet from the ground?

4. Find the square root of 935089.

PROCESS

	9	6	7
	93	50	89
	81		
186	1250		
	1116		
1927	13489		
	13489		

I. Beginning at the ones, point off the number in periods of two figures.

II. As the largest square contained in 93, the first period, is 81, write 81 under 93, and place 9, the square root of 81, above 93.

III. Subtract 81 from 93, and bring down the next period, making 1250 a partial dividend.

IV. Write 18 (twice 9) as a trial divisor. Divide 125 by 18 and take 6 (the quotient) as the next figure of the root. Write 6 above the second period and annex 6 to the trial divisor 18, making the complete divisor 186.

V. Multiply 186 by 6 and subtract the product from 1250, leaving 134. To this annex the next period, 89, making the partial dividend 13489.

VI. Write 192 (twice 96) as the next trial divisor. Divide 1348 by 192 and take 7 as the next figure of the root. Write 7 above the last period and also annex 7 to the trial divisor 192, making the complete divisor 1927.

VII. Multiply 1927 by 7. As there is no remainder, the required root is 967.

5. Find the square root of

a. 56169. *b*. 37249.
c. 140625. *d*. 9.5481.

6. Find the length of the missing side in each of the following right triangles:

a. Hypotenuse, 97 yd.; base, 72 yd.
b. Perpendicular, 91 ft.; hypotenuse, 109 ft.
c. Base, 99 rd.; perpendicular, 20 rd.

7. What is the side of a square steel plate that will contain 150 square inches?

PROCESS

$\sqrt{150} = \sqrt{25} \times 6 = 5\sqrt{6} = 5 \times 2.4495 = 12.2475$ (in.) *Ans.*

Take as the factors of 150, 25 (the largest square) and 6. Write the square root of 25 outside the radical, leaving 6 inside. Take the value of $\sqrt{6}$ from the table.

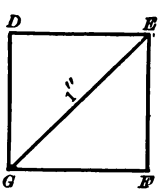
NOTE. — $5\sqrt{6}$ is read "5 times the square root of 6." Since the square-root sign is also called a *radical*, the foregoing expression is sometimes read as "5 radical 6."

8. Give the values of the following:

a. $\sqrt{120} = ?$ *b.* $\sqrt{200} = ?$ *c.* $\sqrt{350} = ?$

9. An acre contains 4840 square yards. How many yards long is a square field containing one acre?

10. Find the side of a square the diagonal of which measures 1 inch.



$$\overline{EF}^2 + \overline{FG}^2 = \overline{EG}^2 = 1.$$

Since $EF = FG$, $2\,\overline{EF}^2 = 1$, and $\overline{EF}^2 = \frac{1}{2}$.

EF is, therefore, equal to $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$.

In order to avoid dividing 1 by 1.4142 ($\sqrt{2}$) the radical is transferred to the numerator of the fraction, by multiplying both terms by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.4142}{2} = .7071 \text{ (in.)}. \quad \textit{Ans.}$$

11. What is the side of a square when its diagonal is 2 feet long?

12. Each side of an equilateral triangle measures 6 inches. Find its altitude.

Square Root of Decimals

Preparatory Exercises

1. What is the square (*a*) of $\frac{3}{10}$? (*b*) Of .3? (*c*) Of $\frac{7}{10}$? (*d*) Of .7? (*e*) How many decimal places in the result in (*b*) and in (*d*)?

2. How many decimal places are there in the product (*a*) of .03 by .03? (*b*) Of .11 by .11?

3. What is the square root (*a*) of 50? (*b*) Of .50? (*c*) Of .5?

4. Give from the table the square root (*a*) of 6. (*b*) Of 60. (*c*) About how many times the former is the latter?

5. (*a*) Can you give from the table the square root of $\frac{30}{100}$? (*b*) How does this compare with the square root of $\frac{3}{10}$?

Observe that $\sqrt{\frac{30}{100}} = \frac{\sqrt{30}}{10}$ and $\sqrt{\frac{3}{10}} = \frac{\sqrt{3}}{\sqrt{10}}$, which means $\sqrt{3} \div \sqrt{10}$, or $1.7321 \div 3.1623$. By changing .3 to .30, this division is avoided.

The square root of .5 is obtained as follows when a table is not available:

PROCESS

Ans. .7071 nearly

	.50	
	49	
1407	10000	
	9849	
14141	15100	
	14141	
14142	95900	

I. Begin at the decimal point, and point off two places to the right, annexing a cipher to supply the second place.

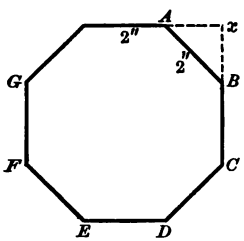
II. Place .7 above .50, and write 49 underneath it. Subtract, and bring down two ciphers (the next period).

III. Since 14 (the trial divisor) is not contained in 10 (the first two figures of the partial dividend), place a cipher in the root, and annex one to the trial divisor, making it 140. Bring down another period, making the partial dividend 10000, etc.

Written Problems

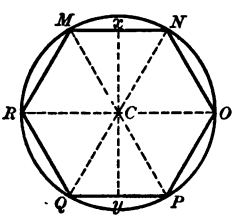
1. A step ladder is 7 ft. 6 in. long. How far apart will be the feet of the ladder when the top is 6 feet above the ground?

2. An octagon having each side 2 inches long is cut from a square piece of paper by cutting off four right triangles. (*a*) What is the length of Ax or xB when AB is 2 inches? (*b*) What is the area of AxB?



3. Two lead sheets of the same thickness are melted together, one being 10 inches square and the other 10.5 inches square, and made into a single square sheet of the same thickness. What is the size of the latter?

4. A hexagon is cut from a circle 4 inches in diameter. (*a*) How long is each side of the equilateral triangle MNC? (*b*) How long is each side of the hexagon? (*c*) Find the length of xC, the altitude of one of the triangles. (*d*) How long is xy?



5. The intensity of a light varies inversely as the square of the distance from the light. In testing an electric light it was found that it gave the same illumination at 9 feet as a candle did at $2\frac{1}{4}$ feet. What was the candle power of the former?

Electric : Candle :: $(9)^2 : (2\frac{1}{4})^2$

6. If a man can just tell the time by his watch when he is 50 feet from a light, how far away from a light 3 times as powerful can he tell it?

Cubes and Cube Roots

$5 \times 5 \times 5$ is written 5^3, the exponent 3 denoting that 5 is taken 3 times as a factor. 5^3 is read "5 *cube*" or "the *third power* of 5."

Sight Exercises

Give answers:

a. $2^3 = ?$ *b.* $3^3 = ?$ *c.* $4^3 = ?$ *d.* $5^3 = ?$

e. $6^3 = ?$ *f.* $7^3 = ?$ *g.* $8^3 = ?$ *h.* $9^3 = ?$

Cube Roots

To denote that the cube root is to be extracted a small 3 is written in the front of the radical sign.

$\sqrt[3]{27} = 3$ is read "the cube root of 27 is 3."

The ordinary method of extracting the cube root is too tedious for business men, who obtain it by using tables, for instance, the following extract from which gives the cubes and the cube roots of numbers to 30:

TABLE

No.	Cube	Cube Root	No.	Cube	Cube Root	No.	Cube	Cube Root
1	1	1.	11	1331	2.2240	21	9261	2.7589
2	8	1.2599	12	1728	2.2894	22	10648	2.8020
3	27	1.4422	13	2197	2.3513	23	12167	2.8439
4	64	1.5874	14	2744	2.4101	24	13824	2.8845
5	125	1.7100	15	3375	2.4662	25	15625	2.9240
6	216	1.8171	16	4096	2.5198	26	17576	2.9625
7	343	1.9129	17	4913	2.5713	27	19683	3.
8	512	2.	18	5832	2.6207	28	21952	3.0366
9	729	2.0801	19	6859	2.6684	29	24389	3.0723
10	1000	2.1544	20	8000	2.7144	30	27000	3.1072

Sight Review Problems

1. If the last three figures of a number are 120, what numbers are certainly factors of it?

2. If a number contains all of the following figures in any order whatever, what two numbers will surely divide it: 1, 3, 0, 5?

3. If one pipe can fill a cistern in 2 hours and another in 3 hours, in what time can both fill it running together?

4. If oranges are bought at the rate of 3 for 5 cents and are sold at the rate of 2 for 5 cents, what is the profit (*a*) on 1 orange? (*b*) On 1 dozen?

5. A starts at 9 o'clock and walks at the rate of 3 miles per hour. At 10 o'clock B starts from the same place and follows A at the rate of 4 miles per hour. At what time will he overtake A?

6. A tank is 4 ft. by 2 ft. by 2 ft. (*a*) How many cubic feet of water will it hold? (*b*) How many pounds of water? (*c*) What part of a ton?

7. In working an example a boy multiplied by 2 instead of dividing by 2, and obtained 16 for his answer. If his work was otherwise correct, what is the right answer?

8. Give the smallest number that (*a*) added to 100 will make the sum a multiple of 17. (*b*) Subtracted from 100 will make the remainder a multiple of 14. Give the nearest number to 100 that is a multiple (*c*) of 28. (*d*) Of 29.

9. How many cubic feet of air are there in a room 24 ft. long, $16\frac{2}{3}$ ft. wide, 9 ft. high?

10. How many strips of carpet $\frac{3}{4}$ yd. wide will cover a floor 18 ft. wide?

11. A person spent $\frac{2}{3}$ of his money and lost $\frac{2}{5}$ of the remainder. He then had $31 left. How much had he at first?

12. Four men take $13\frac{1}{2}$ days to do a piece of work. How many men would be required to do it in 1 day?

13. At the rate of $33\frac{1}{3}$ bushels of corn to the acre, what would be the yield on 3.9 acres?

14. What is the area of a square each side of which measures $5\frac{1}{2}$ yards?

15. How many times can a vessel containing a pint and a half be filled from one containing a gallon and a quarter, and how much is left over?

16. When 4 men do a piece of work in 12 days, give the time required for the same work by (*a*) 6 men. (*b*) 12 men. (*c*) 3 men. (*d*) 18 men.

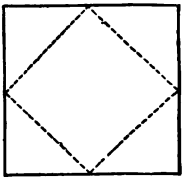
17. When a farmer has sufficient food to keep a herd of 30 cows for 40 days, give the time it will last if the herd is increased by:

a. 30 cows *b.* 10 cows *c.* 20 cows *d.* 50 cows

If the herd is diminished by:

a. 15 cows *b.* 20 cows *c.* 5 cows *d.* 25 cows

18. How many cubical boxes, each of which is 3 ft. 6 in. long, can be placed in a bin 14 ft. by 14 ft. by 14 ft.?



19. When the side of a square is 12 inches, (*a*) how many square inches are there in the area of the inscribed square? (*b*) How long is the diagonal of the inscribed square? (*c*) What is the area of a square whose diagonal is 12 inches?

20. The product of three numbers is 121. One of the numbers is 4 and the other two are equal. Find the equal numbers.

21. How long will it take 25 men to do what 10 men can do in 15 days?

22. When the 4th of July falls on Monday, what day will the 4th of August be?

23. If one side of a rectangular field measures 40 rods, what must be the length of an adjacent side so that the field may contain $7\frac{3}{4}$ acres? (Cancel.)

24. How many square yards more than an acre are there in a field 70 yards square?

25. If a boy walks $2\frac{1}{8}$ miles in 35 minutes, how many miles will he walk in an hour at the same rate?

26. How many cubic feet are there in a cubical box 18 inches in depth?

27. A roller is 22 feet in circumference and 9 feet long. How many square feet of ground does it roll in one revolution?

28. Three pipes can fill a tank in 10 minutes. One can fill it alone in an hour, a second can fill it alone in 30 minutes. How long does the third require to fill it alone?

29. The volume of a cube is 27 cubic yards. What is the entire surface?

30. How many square yards are there in a tight fence 6 feet high enclosing a plot 150 ft. square?

31. A 60-day note for \$350 is discounted at 6%. Give the proceeds.

32. If the discount of a note for \$420 at 5% is \$4.20, in how many days is the note due?

Written Review Problems

1. Reduce to their lowest terms

$$a.\ \frac{336}{720} \qquad b.\ \frac{887}{1184}$$

2. Divide (*a*) 864 by .024. (*b*) .0169 by 5.2.

3. How high is a house casting a shadow 24 feet long, when a tree 70 feet high casts a shadow of 42 feet at the same time?

4. Which is the largest and which is the smallest of the fractions: $\frac{17}{150}$, $\frac{23}{200}$, $\frac{14}{125}$?

5. Simplify:

$$a.\ \tfrac{1}{3} \text{ of } (17\tfrac{1}{2} - 12\tfrac{1}{4}) + \tfrac{2}{7} \text{ of } (3\tfrac{1}{6} - 1\tfrac{1}{3})$$

$$b.\ \frac{5\frac{1}{2} \times 15\frac{3}{11}}{\frac{1}{2} \times \frac{3}{4} \times 18\frac{2}{3}} \qquad c.\ \frac{6\frac{3}{4} + (5\frac{1}{2} \times 3\frac{1}{2}) - 7\frac{1}{4}}{6\frac{2}{5} + 5 - 8\frac{1}{5}}$$

$$d.\ (.26 \times .3 \times .02) \div (.5 \times .01 \times 1.04)$$

6. Find the greatest common divisor (*a*) of 160 and 234. (*b*) Of 135 and 234. (*c*) Of 135 and 160. (*d*) Is there a common factor of 135, 160, and 234?

7. Express as a common fraction the value of

$$.027 \times 2.53 \div .099.$$

8. A bathtub is 6 ft. long, 2 ft. 6 in. wide, and 2 ft. deep. There is an overflow pipe 6 inches from the top. (*a*) How long will water entering the tub at 1 cubic foot per minute require to reach the overflow pipe? (*b*) If the latter is partly stopped and discharges only $\frac{1}{2}$ cu. ft. per minute, how long would it be from the time the water was turned on until it began to flow over the top?

9. Find the per cent of boys and that of girls in a school containing 78 boys and 82 girls.

10. If oranges are bought at the rate of 3 for 5 cents and are sold at the rate of 2 for 5 cents, how many must be sold to make a profit of $3?

11. If a watch that loses .2 min. every hour is set right at noon Jan. 1, what time will it indicate at noon Jan. 11?

12. Express as a decimal the value of each of the following:

a. $\frac{3.2-.5}{.04}$ *b.* $\frac{3.9-.075}{6}$ *c.* $\frac{3.7\times.15}{.37}$

d. $\frac{39.2}{.32}-\frac{52.8}{6600}-\frac{.00403}{.0124}$

13. Find the exact multiple (*a*) of 4.79 that is nearest to 168.88. (*b*) Of .23 that is nearest to 12.379.

14. How many bricks 8 inches long and 4 inches wide are required to cover a floor 17 ft. 4 in. long and 13 ft. 4 in. wide?

15. A schoolroom is 40 ft. square by 12 ft. high. If it contains 39 pupils and a teacher, (*a*) how many cubic feet of air space are there for each? (*b*) How many square feet of floor space?

16. Find the simple interest on 417.53 francs for $2\frac{1}{2}$ years at $5\frac{1}{2}$ %.

17. A man gives 612.45 francs to 36 persons. He gives 13.60 francs each to 15 of them and divides the balance equally among the others. How much does each of the latter receive?

18. A man divides a farm of 387 acres among his three sons. To the first he gives an 8-acre field, to the second he gives a field 17 acres larger. He then divides the remainder equally among the three. How many acres does each receive, including the two fields?

19. How many packages 10 in. long, $3\frac{3}{4}$ in. wide, and $3\frac{1}{2}$ in. deep can be packed in a case whose inside measurements are 2 ft. 6 in., 3 ft. 4 in., and 3 ft. 6 in., respectively?

20. A man borrows \$200, and at the end of each year he gives \$50 to pay the interest at 6 % and to reduce the principal. How much does he owe at the end of 4 years?

21. How many blocks 2 in. by 3 in. by 5 in. will fill a case measuring on the inside 3 ft. by $2\frac{1}{2}$ ft. by 4 ft.?

22. Three men rent a field for \$60. One puts into it 28 sheep for 2 months, another 12 sheep for 8 months, the third 48 sheep for 1 month. How much should each pay?

23. A man borrows \$600. At the end of each year he pays \$200 to reduce the principal and to pay the interest due at the rate of 6 per cent. How much does he owe at the end of three years, after he has made the regular payment of \$200?

24. Find the proceeds of the following note, discounted at 6 %, Oct. 14, 1912:

ACCOTINK, VA., Sept. 20, 1912

Ninety days after date I promise to pay to the order of Magnus Schuler, Three Hundred 00/100 Dollars, value received, at the Pohick National Bank.

JOHN MCWILLIAMS

25. A owes B \$45 and C \$30; B owes C \$15 and D \$50; C owes D \$45; D owes A \$30 and B \$20. What will be the simplest way of settling up?

26. What number multiplied by 4669 gives the same result as 1334 multiplied by 1771?

27. A note for \$360 was discounted at 6% on July 7, the proceeds being \$357.78. (*a*) In how many days is the note due? (*b*) On what date is it due? (*c*) If it was drawn for 90 days, on what date was it made?

28. When it requires 9 days to finish a certain piece of work, how many days will be required if the work is increased one third and the number of men is decreased one fourth?

29. What is the weight of the water in a tank 13 feet long, 4 yards wide, when the water is 18 inches deep?

30. On each day from Monday to Friday (inclusive) of each week, some workmen repaired 27 yards of a road 7899 yards long; on each Saturday they repaired 12 yards; no work was done on Sunday. If they began work on Monday, the 6th of the month, on what date did they finish?

31. A man increases his fortune each year by $\frac{1}{8}$ of its value at the beginning of the year. At the end of the year he deducts \$1800 from his profits for personal expenses. At the end of 1914 he is worth \$24,600 after the deduction. What was he worth (*a*) at the beginning of 1914? (*b*) At the beginning of 1913? (*c*) At the beginning of 1912?

32. The capacity of a hall is 300 grown persons or 420 children. If 120 grown persons are present, how many children may be admitted?

33. Find the weight of an iron plate 24 ft. long, 15 ft. wide, and 18 in. thick, when iron is 7.6 times as heavy as water, and a cubic foot of water weighs 1000 oz.

34. Find how much $\frac{16}{25}$ is increased or diminished: (*a*) when 1 is added to each member; (*b*) when 1 is subtracted from each member.

Measurements

Written Problems

1. A room is 18 feet long, 12 feet wide, 9 feet high.

a. Find the number of square feet in the floor.

b. How many square yards of carpet will cover the floor?

c. Find the number of square feet in the ceiling.

d. How many running feet of baseboard are there in four sides of the room, deducting 3 feet for the space taken by a door?

e. How many square feet of baseboard are there in the room, when the baseboard is one foot high?

2. A paper cracker box is 8 inches long, 3 inches wide, 3 inches high.

a. How many square inches of paper are required for each of the square ends?

b. How many square inches of paper are required for each of the four long faces?

c. How many square inches of paper are required for all six faces, making no allowance for overlapping?

d. How many crackers 3 inches square will it hold, if each cracker is $\frac{1}{4}$ inch thick?

3. How many square feet of wall paper are there in a roll of paper 18 yards long, 18 inches wide?

4. At \$1.50 per square yard, what will be the cost of the linoleum covering a kitchen 15 feet square?

5. A fence on the front of a lot is 25 feet long and 6 feet high. How many square feet does it contain?

SECTION VII

INDUSTRIAL APPLICATIONS

The farmer, the mechanic, the storekeeper, the householder, is frequently placed at a disadvantage by his inability to compute simple problems involving the finding of the areas, volumes, etc.

Even when his calling does not require him to make such calculations regularly, he should be able to test, at least approximately, the correctness of bills based upon quantity of materials furnished and the like.

He should know the lines that must be measured to obtain the data required before beginning his calculations, the allowances to be made, etc.

In order to avoid waste in making purchases, it is advantageous for the housewife to be able to determine the quantity of material required for a garment, the number of yards of ribbon to trim a hat, etc.

Some of the exercises of this section may be profitably replaced by mensuration examples drawn from the industries of the vicinity, and by problems arising in the home experiences of the pupils, in the sewing and shop-work classes, etc.

Other supplementary work should consist of the determination of areas and volumes from data obtained by measurements made by the pupils. The school site, the classroom, a neighboring field, etc., will furnish opportunities.

Diagrams carefully drawn to scale should be occasionally employed in these exercises.

Board Measure

Board measure is used in selling sawed lumber, including boards, beams, joists, rafters, planks, etc.

> *Number of board feet = Length in feet × Width in feet × Thickness in inches.*

Thus, the number of board feet in a board 12 feet long, 4 inches wide, and 1 inch thick or less, is $12 \times \frac{1}{3} \times 1$.

The unit of board measure is the *board foot*, which is 1 foot long, 1 foot wide, and 1 inch thick. Boards less than 1 inch thick are considered as having a thickness of an inch.

Table

Lumber dealers use a table in determining the board feet in a given piece of timber. The following is a portion of such a table.

Size in Inches	Length in Feet							
	10	12	14	16	18	22	26	30
2 × 4	$6\frac{2}{3}$	8	$9\frac{1}{3}$	$10\frac{2}{3}$	12	$14\frac{2}{3}$	$17\frac{1}{3}$	20
2 × 6	10	12	14	16	18	22	26	30
2 × 8	$13\frac{1}{3}$	16	$18\frac{2}{3}$	$21\frac{1}{3}$	24	$29\frac{1}{3}$	$34\frac{2}{3}$	40
2 × 10	$16\frac{2}{3}$	20	$23\frac{1}{3}$	$26\frac{2}{3}$	30	$36\frac{2}{3}$	$43\frac{1}{3}$	50
2 × 14	$23\frac{1}{3}$	28	$32\frac{2}{3}$	$37\frac{1}{3}$	42	$51\frac{1}{3}$	$60\frac{2}{3}$	70
$2\frac{1}{2} \times 12$	25	30	35	40	45	55	65	75
$2\frac{1}{2} \times 14$	$29\frac{1}{6}$	35	$40\frac{5}{6}$	$46\frac{2}{3}$	$52\frac{1}{2}$	$64\frac{1}{6}$	$75\frac{5}{6}$	$87\frac{1}{2}$
$2\frac{1}{2} \times 16$	$33\frac{1}{3}$	40	$46\frac{2}{3}$	$53\frac{1}{3}$	60	$73\frac{1}{3}$	$86\frac{2}{3}$	100
3 × 6	15	18	21	24	27	33	39	45
3 × 8	20	24	28	32	36	44	52	60
3 × 10	25	30	35	40	45	55	65	75
4 × 6	20	24	28	32	36	44	52	60
6 × 10	50	60	70	80	90	110	130	150

Sight Exercises

1. Give the number of board feet in the following boards, each 1 inch thick:

a. 14 ft. long, 1 ft. wide.
b. 10 ft. long, 6 in. wide.
c. 12 ft. long, 3 in. wide.
d. 18 ft. long, 4 in. wide.
e. 16 ft. long, 9 in. wide.
f. 12 ft. long, 8 in. wide.
g. 16 ft. long, 4 in. wide.
h. 12 ft. long, 6 in. wide.
i. 18 ft. long, 9 in. wide.
j. 14 ft. long, 3 in. wide.

2. Give the number of board feet in the following planks, rafters, etc., each 2 in. thick:

a. 14 ft. long, 9 in. wide.
b. 10 ft. long, 6 in. wide.
c. 12 ft. long, 4 in. wide.
d. 18 ft. long, 6 in. wide.
e. 12 ft. long, 6 in. wide.
f. 16 ft. long, 8 in. wide.
g. 10 ft. long, 9 in. wide.
h. 18 ft. long, 4 in. wide.

3. Give the number of board feet in the following, each being 12 ft. long:

a. 2 in. by 4 in.
b. 2 in. by 6 in.
c. 3 in. by 4 in.
d. 4 in. by 4 in.
e. 3 in. by 10 in.
f. 3 in. by 8 in.
g. $5\frac{1}{2}$ in. by 6 in.
h. 8 in. by 10 in.
i. 3 in. by 6 in.
j. 4 in. by 5 in.
k. 2 in. by 10 in.
l. 3 in. by 12 in.

Written Problems

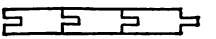
1. At \$30 per M (thousand) find the cost of 120 planks 16 ft. long, 8 in. wide, 2 in. thick.

$$\text{Cost} = \frac{\$30 \times 120 \times 16 \times \frac{2}{3} \times 2}{1000}$$

2. What is the weight of a piece of timber 16 ft. long, 8 in. wide, 2 in. thick at 36 pounds to the cubic foot?

3. How many board feet are there in 100 rafters 2 in. by 7 in., 12 ft. long?

Matched Boards

Boards used for floors and ceilings are "tongued and grooved," the tongue of one board being fitted into the groove of the next one.  These boards are generally 3 inches or 6 inches wide, including the tongue. When laid, each board covers a strip $\frac{1}{2}$ inch less in width.

4. (*a*) How many matched boards 16 ft. long, 3 in. wide will be required for a floor 16 ft. long, $12\frac{1}{2}$ ft. wide when the tongue is $\frac{1}{2}$ inch wide?

$$\text{Number} = 12\tfrac{1}{2} \text{ ft.} \div 2\tfrac{1}{2} \text{ in.}$$

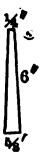
(*b*) Find the cost of the boards at \$36 per M, considering them as 3-inch boards.

$$\text{Cost} = \frac{\$36 \times \text{number of boards} \times 16 \times \frac{1}{4}}{1000}$$

5. (*a*) How many square feet are there in the ceiling of a porch 27 ft. 6 in. long, 8 ft. wide? (*b*) How many feet of boards 3 inches wide are required when $\frac{1}{2}$ inch is lost in matching? (*c*) How many feet of boards 6 inches wide are required when $\frac{1}{2}$ inch is lost in matching?

Siding

Boards used for the outside covering of a house are generally beveled. Siding is $\frac{1}{4}$ in. thick at the top and $\frac{5}{8}$ in. at the bottom. It is generally 6 inches wide and 12 or 16 ft. long. It is laid horizontally, each row overlapping one fourth of the one below it. A 6-inch board, therefore, covers only $4\frac{1}{2}$ inches of surface.



6. (*a*) How many rows of siding will be required for one side of a building 18 feet high?

$$18 \text{ ft.} \div 4\tfrac{1}{2} \text{ in.}$$

(*b*) When the boards are 12 feet long how many boards will be needed for each row in the side of a building measuring 36 feet? (*c*) How many boards will be needed for a side of a building 36 feet long 18 feet high, deducting for 6 windows, each 6 ft. by 3 ft.?

Shingles

Shingles are usually 16 and 18 inches long, but each row overlaps the one below, leaving only 4 or 5 inches exposed. They range in width from $2\frac{1}{2}$ to 14 or more inches, but the unit of measure is 4 inches. They are put up in bundles of a quarter of a thousand, a "thousand" meaning the equivalent of 1000 shingles 4 inches wide.

7. (*a*) When shingles are laid 4″ "to the weather," how many 4-in. shingles cover a square foot? (*b*) How many are required when laid 5″ "to the weather"?

In (*a*) each covers $4'' \times 4''$; in (*b*) $4'' \times 5''$.

8. How many square feet will 1000 shingles cover when they are laid (*a*) 4 inches "to the weather"? (*b*) 5 inches "to the weather"?

9. (*a*) How many bundles of shingles (250) will be required for a roof 40 ft. by 25 ft. when they are laid 4 inches to the weather? (*b*) How many slates 16 inches wide laid 10 inches to the weather would be needed?

10. How many thousand shingles will be required for the side of a house 36 ft. long, 18 ft. high, deducting for 6 windows each 6 ft. by 3 ft., if they are laid 5 inches to the weather and 112 shingles are allowed for waste?

Laths

Ordinary laths are 4 ft. long, $1\frac{1}{2}$ in. wide, $\frac{1}{4}$ in. thick. They are sold in bunches of 100. Being laid $\frac{1}{2}$ in. apart, 1 lath is required for a space 4 ft. by 2 in.

11. How many laths are required for 100 sq. yd. when 1 lath is needed for a space 4 ft. by 2 in., and no allowance is made for waste?

12. (*a*) How many square yards of plastering are there in the walls and the ceiling of a room 15 ft. long, 12 ft. wide, 9 ft. high deducting for 2 doors, each 3 ft. by $7\frac{1}{2}$ ft., and 2 windows each 3 ft. by 6 ft.? (*b*) Adding $2\frac{1}{2}\%$ for waste, how many laths are required?

13. Find the total cost of the materials required to lath and plaster 100 square yards, at the following quantities and prices:

10 bushels of lime at 40¢ per bushel.
$1\frac{1}{2}$ cubic yards of sand at 75¢ for $1\frac{1}{4}$ cu. yd.
2 bushels of hair at 40¢ per bushel.
100 pounds of plaster of Paris for 50 cents.
1400 laths at $2.75 per M.
10 pounds of nails at 14¢ per pound.

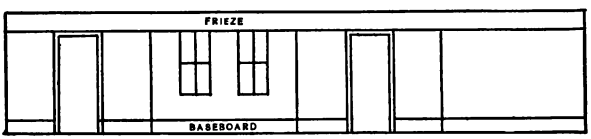
Painting

14. How many gallons of paint will be required for 3 coats on two sides of a house 36 ft. deep and 18 ft. high, if the first coat takes 1 gal. for 60 sq. yd., the second 1 gal. for 72 sq. yd., and the third coat 1 gal. for 90 sq. yd.?

15. The front of a house is 24′ wide, 18′ high to the eaves, and 27′ to the ridgepole. (*a*) Find the number of sq. yd. in the surface, deducting for a door $7\frac{1}{2}' \times 3'$, 3 windows, each $6' \times 3'$, and 1 window $4' \times 3'$.

Wall Paper

Wall paper is generally 18 inches wide. It is sold by the single roll of 8 yards, or the double roll of 16 yards. Borders are either 9 inches wide or 18 inches wide. A wide border at the top of a wall is called a *frieze*.



16. A room is 15′ long, 12′ wide, and $10\frac{1}{2}$′ high. (*a*) How many sq. ft. of baseboard 1′ wide does it contain, deducting for 2 doors 3′ wide? (*b*) How many rolls of border 8 yd. long and 18 in. wide are required for the frieze? (*c*) How many single rolls of paper are required to cover the walls above the baseboard and below the frieze, deducting for 2 doors, each 9′ × 3′, and 2 windows, each 6′ × 3′, and allowing 12 sq. ft. for waste?

17. How many square yards of plastering are required for the walls and the ceiling of the same room, the plaster extending to the bottom of the baseboard?

18. How many board feet of 3-inch matched flooring are required, allowing $\frac{1}{2}$ inch for waste on each board?

Carpeting

19. (*a*) How many square yards of carpet are required to cover a floor $5\frac{1}{3}$ yards wide, $6\frac{3}{4}$ yards long? (*b*) Find the number of running yards of carpet employed to cover the floor when the carpet is $\frac{3}{4}$ yard wide.

20. At \$1.25 per yard laid upon the floor, what will be the cost of carpeting a floor 15 ft. long, 12 ft. wide, when the carpet is 27 inches wide?

21. How many yards of carpet 1 yd. wide must be purchased for a floor 20 ft. 3 in. long, 16 ft. wide, when the strips run lengthwise?

When the strips run lengthwise, $5\frac{1}{4}$ strips are required, but 6 strips must be purchased. The length of the room being $6\frac{3}{4}$ yd., the number of yards is $6\frac{3}{4} \times 6$, etc.

22. How many yards of carpet 27 in. wide will be needed for the floor of a room 20 ft. 6 in. long, 16 ft. wide, when the strips run crosswise?

Number of strips: $20\frac{1}{2}$ ft. $\div$ 27 in. $=$ 246 in. $\div$ 27 in. $= 9\frac{1}{9}$; 10 strips must be used. Length of each strip, 16 ft., or $5\frac{1}{3}$ yd. Quantity of carpet to be bought: $5\frac{1}{3}$ yd. $\times$ 10, etc.

23. A floor is 20 ft. 3 in. long, 16 ft. wide. How many yards of carpet are needed (*a*) when the carpet is 1 yard wide and the strips run crosswise? (*b*) Find the number of yards of carpet 27 inches wide that must be bought when the strips run lengthwise. (*c*) When they run crosswise.

Matching Patterns. — In "making" a carpet for a given floor, the strips are sewed together in such a way as to "match the patterns." This frequently requires that a portion of each strip except the first be cut off, the amount varying according to the pattern.

24. How many yards of carpet, 27 inches wide, are required for the floor of a room 20 ft. 3 in. long, 16 ft. wide, when 4 inches are wasted on each strip except the first, and the strips run lengthwise?

25. How many inches must be cut off every strip except the first to match the pattern when the first strip is 20 ft. 3 in. long, and the pattern is repeated (*a*) every 9 inches? (*b*) Every 12 inches? (*c*) Every 8 inches?

Deductions for Openings

In building walls by the cubic yard; in painting, plastering, etc., by the square yard, contractors do not make a full allowance for openings. In some cases no deduction is made for openings below a certain size; in other cases one half of the area of the openings is deducted.

In ascertaining the quantity of material required, the actual surface or volume is used, due consideration, however, being given to material necessarily wasted.

26. (*a*) At 30¢ per square yard, find the cost of plastering the walls and the ceiling of a room 15 ft. long, 12 ft. wide, and $10\frac{1}{2}$ ft. high, making one half allowance for 2 doors, each 9 ft. by 3 ft., and 2 windows, each 6 ft. by 3 ft. (*b*) Find the cost of tinting the walls and the ceiling at 8¢ per square yard, making the same allowance. (*c*) At 35 cents per square yard, find the cost of painting the baseboard, which is 1 ft. wide, the doors and the windows, making no allowance in the last for the space occupied by the glass.

27. The outside dimensions of the walls of a cellar are 36 ft. by 24 ft. (*a*) How many square feet remain for the floor of the cellar, if the walls are 2 ft. thick? (*b*) How many square feet are occupied by the walls? (*c*) If the walls are 9 ft. high, how many cubic yards of material do they contain, if there is one opening 9 ft. by 4 ft. and three openings each 3 ft. square? (*d*) How many tons of stone are required, if one ton is sufficient for a perch of $16\frac{1}{2}$ cu. ft.? (*e*) Find the cost of building the walls at $3 per perch of 22 cu. ft., if the walls are measured on the outside and one half allowance is made for the openings.

The walls are considered by the contractor as equivalent to a wall (36 ft. + 24 ft. + 36 ft. + 24 ft.) long, 9 ft. high, 2 ft. thick.

28. A finished road consists of 3 inches of fine broken stone laid on 12 inches of coarser material. (*a*) How many cubic yards of each must be spread before rolling for a mile of road 27 feet wide, assuming that the steam roller will compress it into three fourths of the space it occupied when loose? (*b*) How many cart loads will be required at $\frac{5}{8}$ cu. yd. to the load?

29. A printer has an order for 1500 cards $3''$ by $2\frac{1}{2}''$. He has 25 sheets of pasteboard measuring $24''$ by $18''$. What is the largest number of cards he can get out of this material?

30. Two rectangular cisterns, with lids, are to be made of sheet iron. One measures $12' \times 8' \times 6'$, the other $16' \times 9' \times 4'$. (*a*) Find the number of square feet of material required for each, making no allowance for seams. (*b*) What is the capacity of each in cubic feet?

31. Find the difference in the number of cubic inches between a solid $12'' \times 8'' \times 6''$ and one $14'' \times 10'' \times 8''$.

32. How many board feet of inch boards will be required for a covered box having inside dimensions $12'' \times 8'' \times 6''$?

33. A window sash whose outside dimensions are $5' \times 3'\ 6''$ contains 4 panes of glass of the same size. The frame is $2''$ wide and the panes are separated from each other by strips $1''$ wide. Make a drawing showing the dimensions of each pane.

34. A stair carpet covers 18 steps, $10''$ tread and $7''$ rise, with $18''$ extra at both the bottom and the top of the staircase. How many yards of carpet are required?

35. A merchant imports 8 pieces of cloth, 36 yards to the piece. How many square yards of cloth are there if it is 1 meter wide?

Dimensions of a Triangle

Preparatory Exercises

1. Take four narrow strips of cardboard (Fig. 1). Make them into a quadrilateral by sticking a pin through each two strips that form a corner. Does the quadrilateral retain its shape under pressure?

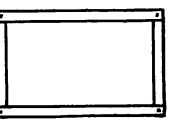


FIG. 1.

2. Form three narrow strips of cardboard into a triangle (Fig. 2). Does it retain its shape under pressure?

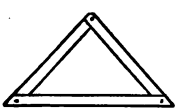


FIG. 2.

There can be only one triangle having sides of given lengths. An indefinite number of quadrilaterals can be formed of sides of given lengths.

As will be seen, the area of a triangle can be calculated when the lengths of the sides are given. The area of a parallelogram or other quadrilateral cannot be ascertained by merely measuring the lengths of the sides. The inclination of the adjacent sides must be known, or the perpendicular distance between two parallel sides, or the length of a diagonal, etc.

Written Exercises

1. The triangle *MNO* (Fig. 3) is right-angled at *M*. The sides *MN* and *MO* measure, respectively, 30 ft. and 40 ft. (*a*) Find its area. (*b*) Find the hypotenuse *NO*. (*c*) Considering *NO* as the base, find the altitude *Mz*, using the area already obtained in (*a*) and the length of *NO* found in (*b*).

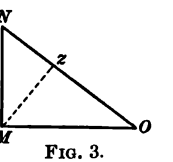


FIG. 3.

NOTE. — In finding the area, either *MN* or *MO* is taken as the base, the other then being the perpendicular.

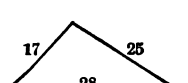


FIG. 4.

2. Find the area of a steel plate (Fig. 4) whose sides measure 17 inches, 25 inches, and 28 inches, respectively.

When the altitude of a triangle cannot be measured, the area may be found from the length of the sides, as follows:

PROCESS

17
25
28
2)70
35 − 17 = 18
35 − 25 = 10
35 − 28 = 7

The half sum of the sides is 35. From this each side is subtracted separately, giving remainders 18, 10, 7. The square root of the continued product of the half sum and the three remainders is the required area.

Area in square rods $= \sqrt{35 \times 18 \times 10 \times 7} = 210.$

Ans. 210 sq. rd.

In determining the value of $\sqrt{35 \times 18 \times 10 \times 7}$ these numbers may be factored, giving $\sqrt{7 \times 5 \times 9 \times 2 \times 5 \times 2 \times 7}$. This result may be written $\sqrt{7^2 \times 5^2 \times 3^2 \times 2^2}$, which is equal to $7 \times 5 \times 3 \times 2$.

In factoring 18, select 9, which is a square, as one factor.

3. The following is a right triangle. Find its area, however, by the method just given. The sides are 20 yd., 21 yd., 29 yd.

4. Find the area of each of the following triangles: (*a*) 13 in., 14 in., 15 in. (*b*) 13 ft., 20 ft., 21 ft.

5. Draw two isosceles triangles having sides as follows: (*a*) 5 in., 5 in., 6 in. (*b*) 5 in., 5 in., 8 in. (*c*) Find the area of each.

6. (*a*) From the table of square roots find the altitude of an equilateral triangle having 2-inch sides. (*b*) Find the area by multiplying the altitude by one half the base. (*c*) Find the area by the method given in example 2.

7. Compare the area of a 4-inch equilateral triangle with that of a 2-inch equilateral triangle.

8. (*a*) What is the diameter of a circle whose area is 616 square rods? (*b*) The circumference?

PROCESS

First obtain the radius by using the formula

$$3\tfrac{1}{7}\,R^2 = 616$$

$$R^2 = 616 \div 3\tfrac{1}{7} = 616 \times \tfrac{7}{22} = 196$$

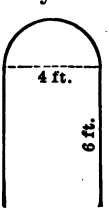
$$R^2 = 196;\ R = \sqrt{196} = 14$$

When the radius is 14 rods, the diameter is twice 14 rods, or 28 rods.

9. The area of a circle is 12,474 sq. ft. Find (*a*) the radius. (*b*) The diameter. (*c*) The circumference.

10. What will be the area of the top of the stump made by sawing down a tree at a point where its circumference is 44 feet?

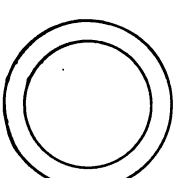
When the circumference is given, the area can be obtained by multiplying $\frac{1}{4}$ the circumference by $\frac{1}{2}$ the diameter.



11. Find the area of a window the lower part of which is a rectangle 4 ft. wide, 6 ft. high, and the upper part a semicircle.

12. A circular window is 7 feet in diameter. Find its area.

13. Find the area of a circular walk 7 feet wide surrounding a plot 28 feet in diameter.

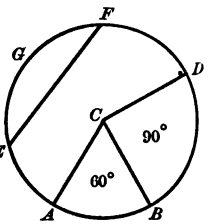


Find the difference between the area of the outer circle and that of the inner one.

14. Find the cost of resilvering a circular mirror 1 ft. 9 in. in diameter at 32 cents per square foot.

Sectors and Segments

A portion of a circle bounded by two radii and an arc is called a *sector.*



ABC is a sector of 60°; BCD is a sector of 90°, C being the center of the circle.

A sector of 90° is called a *quadrant;* one of 60° is called a *sextant.* What is the name of a sector of 180°?

A *segment* is a portion of a circle bounded by an arc and its chord. EFG is a segment.

Sight Exercises

1. In a circle whose diameter is 7 in., give (*a*) the length of the circumference. (*b*) The length of a semicircumference. (*c*) The length of an arc of 90°. (*d*) The length of an arc of 60°. (*e*) The length of an arc of 120°.

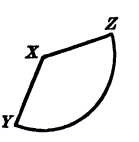
2. In a circle whose diameter is 14 inches, give the area (*a*) of the circle. (*b*) Of a semicircle. (*c*) Of a quadrant. (*d*) Of a sextant. (*e*) Of a sector of 120°.

Written Exercises

1. Find the area of a sextant, when the radius of the circle is 14 inches.

2. Find the area of a quadrant of a circle whose radius is 21 feet.

3. In the sector XYZ, the length of the arc is 33 yards, and the sides XY and XZ measure 14 yards each. Find the area.



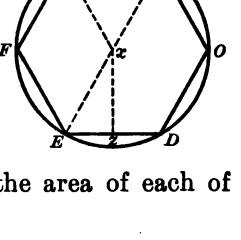
Since $Area\ of\ circle = \frac{1}{2}(R \times circumference)$

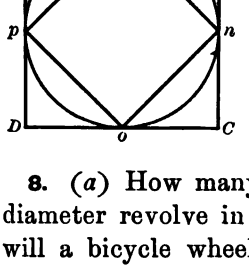
$Area\ of\ sector = \frac{1}{2}(R \times arc)$.

4. In a circle having a diameter of 70 inches, find the length of an arc (*a*) of 180°. (*b*) Of 90°. (*c*) Of 72°. (*d*) Of 60°.

5. What is (*a*) the area of a sector of 72°, the sides being 35 inches long? (*b*) The length of the arc?

6. The hexagon *ABCDEF* is inscribed in a circle having a radius of 7 ft. (*a*) What is the length of *AB*, the chord of an arc of 60°? (*b*) Find the length of *xz*, the altitude of one of the six equilateral triangles composing the hexagon. (*c*) Find the area of each of the six triangles.





7. The square *ABCD* has sides 14 inches long. (*a*) Find the area. (*b*) Find the area of the square *mnop*. (*c*) Find the area of the circle inscribed in *ABCD*.

8. (*a*) How many times will a wagon wheel $3\frac{1}{2}$ feet in diameter revolve in going a mile? (*b*) How many times will a bicycle wheel 28 inches in diameter revolve in going a mile?

9. If a steam roller is 7 feet high and 8 feet long, how many square feet of ground will it roll at each revolution?

10. Four circles each 3 inches in radius are cut from a square piece of cardboard. (*a*) Find the area of the smallest piece of cardboard that will answer, and (*b*) the area of the four circles.

11. The lower part of a window 7 feet wide, is 10 feet long. The upper part is a semicircle. What is the area of the window?

12. The outer diameter of an iron pipe 7 feet long is 14 inches, and the iron is 2 inches thick. (*a*) How many cubic inches of iron are there in the pipe? (*b*) How many cubic feet?

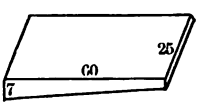
The volume of the pipe is the difference between the volume of two cylinders, both 84 inches high, and having diameters of 14 inches and 10 inches respectively.

Volume in cubic inches $= (7^2\pi - 5^2\pi) \times 84$; or $(7^2 - 5^2) \times 3\frac{1}{7} \times 84$.

Number of cubic feet $= \frac{(7^2 - 5^2) \times 3\frac{1}{7} \times 84}{1728}$ Cancel.

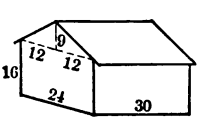
13. The water in a swimming tank 60 ft. long and 25 ft. wide is 3 ft. deep at one end and 7 ft. at the other. How many gallons does it contain at $7\frac{1}{2}$ gallons to the cubic foot?

Consider as the base of the prism one side of the tank, which is a trapezoid having two parallel sides of 7 ft. and 3 ft., respectively, with a perpendicular distance between them of 60 ft. The altitude is 25 ft.



14. A two-story and attic house without a cellar is 24 feet wide, 30 feet long, 16 feet high to the eaves and 25 to the ridge pole. (*a*) Find the number of cubic feet in the attic. (*b*) The number of cubic feet in the rest of the house. (*c*) The outside surface.

The house is a prism, the ends being the bases. The attic may be considered a triangular prism 30 ft. in altitude, the base being a triangle with a base of 24 ft. and an altitude of 9 ft.



15. A builder agrees to erect it at the rate (*a*) of 60 cents per square foot of outside surface, or at the rate (*b*) of 6 cents per cubic foot for the cubic contents of the attic, and 13 cents per cubic foot for the cubic contents of the rest of the house. Find the cost of the house at each rate.

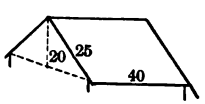
16. The roof of this house extends one foot beyond the walls on all sides. (*a*) What are the dimensions of each half of the roof? (*b*) What is the total area of the roof? (*c*) At 9 shingles to the square foot, how many shingles will be required?

17. How many cubic feet of ensilage will a rectangular silo contain, the interior dimensions of the base being 14 ft. by 14 ft., and its height 30 ft.?

18. (*a*) Find the cubical contents of a cylindrical silo 14 feet in diameter (interior) when filled to the height of 30 feet. (*b*) Find the weight of the contents at 40 pounds to the cubic foot. (*c*) When a cow is fed 35 pounds per day, how many cows will it supply for 176 days? (*d*) At the rate of 14 tons to the acre how many acres of ground will furnish the necessary fodder to fill the silo to the height of 30 ft.?

19. Each half of the roof of a house is 40 ft. by 25 ft. The top of the roof is 20 ft. above the bottom. How many cubic feet of snow are there on the roof after a 6-inch fall?

Note that the snow on a sloping roof is the same in volume as it would be on the space covered by the roof if the latter were removed.



In the determination of the quantity of iron, etc., in rods, beams, etc., business men use the following formula:

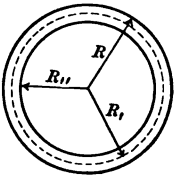
Volume = *Length* × *Area of cross section.*

in which "cross section" is substituted for the word "base," and "length" for "height."

20. Find the number of cubic feet in 4 rods each 18 ft. long with a cross section 2 inches square.

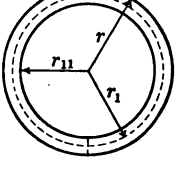
21. How many cubic inches are there in a circular rod 5 ft. 4 in. long and $1\frac{3}{4}$ inches in diameter?

22. A ring is made of an iron bar having a rectangular cross section 2 inches square. (*a*) When the interior diameter of the ring is 6 inches, what is the exterior diameter? (*b*) What is the radius (R) of the outer circle? (*c*) What is the radius of the inner circle (R_{II})? (*d*) How many times π is $R^2\pi - R_I^2\pi$? (*e*) How many square inches are there in the surface of the top of the ring? (*f*) What is the depth of the ring? (*g*) Find the number of cubic inches in the ring.



23. This ring is made from a bar of iron having a cross section of 2 inches square and equal in length to the circumference of a circle whose radius is R_I in the diagram, the distance from the center of the enclosed circle to the middle of the ring. (*a*) How long is the bar? (*b*) How many cubic inches does it contain?

24. (*a*) How long a circular rod 2 inches in diameter will be required to make an "anchor" ring having an interior diameter of 6 inches, the length being equal to the circumference of a circle having a diameter halfway between that of the outer and the inner diameter of the ring? (*b*) What is the area of a cross section of the rod? (*c*) How many cubic inches of iron are there in the ring? (*d*) Find the ratio between the area of a cross section of this ring and that of the ring made from the bar. (*e*) Find the ratio between the quantity of iron in this ring and the quantity in the other.



Reading a Working Drawing

The lines on the paper on which the following diagrams are drawn, are $\frac{1}{8}''$ apart.

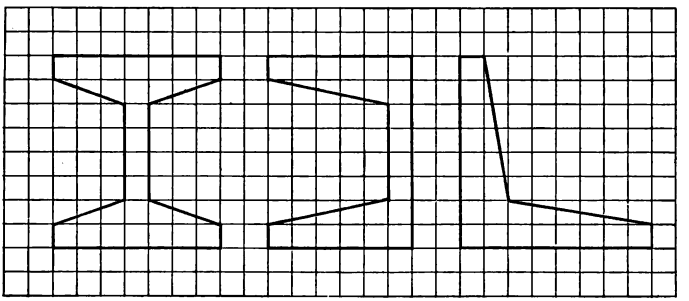


FIG. 1. FIG. 2. FIG. 3.

Sight Exercises

When the scale is $16''$ to $1''$, (*a*) how high is each of the beams whose cross sections are shown above? (*b*) If the cross section in Fig. 1 is divided into four trapezoids and a rectangle, what are the dimensions of the latter?

Written Exercises

1. (*a*) Find the area in square inches of the cross section represented in Fig. 1. (*b*) Find the number of cubic feet in a beam 18 ft. long. (*c*) Find its weight at 480 pounds to the cubic foot.

2. (*a*) Find the area of the cross section of a channel beam (Fig. 2). (*b*) Find the number of cubic feet in a beam 16 ft. long. (*c*) Find its weight.

3. (*a*) Find the area of the cross section of an angle beam (Fig. 3). (*b*) Find the number of cubic feet in a beam 12 feet long. (*c*) Find its weight.

Measurements

Sight Review Problems

1. A roll of oilcloth contains 30 square yards; give its length (*a*) when the oilcloth is 2 yards wide. (*b*) When it is $1\frac{1}{2}$ yards wide.

2. Give the length (*a*) of a strip of carpet $\frac{3}{4}$ yd. wide, containing 30 square yards. (*b*) Of one 27 in. wide containing 27 square yards. (*c*) Of one 27 in. wide containing 30 square yards.

3. How many running feet of boards will contain 1200 sq. ft., (*a*) when the boards are 6 in. wide? (*b*) 4 in.? (*c*) 8 in.? (*d*) 9 in.?

4. Find, as a fraction of a square foot, the top surface of a brick (*a*) 8 in. long, 4 in. wide. (*b*) How many square feet will 900 bricks cover when each covers $\frac{2}{9}$ sq. ft.? (*c*) How many bricks, each covering $\frac{2}{9}$ sq. ft., are required to cover a surface of 1000 sq. ft.?

5. A strip of land $\frac{1}{2}$ yard wide contains an acre (4840 sq. yd.); (*a*) how long is it? (*b*) Give the length of a strip 18 inches wide, which contains an acre.

6. (*a*) What fraction of a square foot is covered by a shingle when it covers a space 4 in. by 4 in.? (*b*) How many shingles are required to cover 100 sq. ft. when each covers $\frac{1}{9}$ sq. ft.? (*c*) When each covers a space $\frac{5}{12}$ ft. by $\frac{1}{8}$ ft.?

7. (*a*) What is the volume of a brick whose dimensions are $\frac{2}{3}$ ft. by $\frac{1}{3}$ ft. by $\frac{1}{6}$ ft.? (*b*) How many bricks are there to the cubic foot if each brick contains $\frac{1}{27}$ cu. ft.? (*c*) How many cubic inches are there in a paving brick whose dimensions are 8 in. by 4 in. by $2\frac{1}{2}$ in.?

Written Review Problems

1. A strip of ground is 5 miles long and 9 inches wide. Find its area (*a*) in square yards. (*b*) In acres.

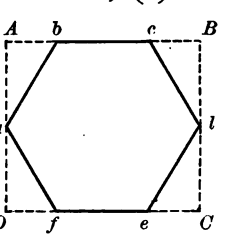
As there are 1760 yards in a mile, the area in square yards will be $1760 \times 5 \times \frac{1}{4}$. As there are 4840 square yards to an acre, the area in acres will be $1760 \times 5 \times \frac{1}{4} \div 4840$.

2. How many miles does a team travel in plowing 1 acre when the plow makes a furrow 9 inches wide?

Omitting from consideration the distance traveled in making turns, the plowed ground may be considered as a rectangle 9 inches wide, having an area of 1 acre. Changing the area to 4840 sq. yd., and the width to $\frac{1}{4}$ yd., the length in yards is $4840 \div \frac{1}{4}$, and the length in miles is $4840 \div \frac{1}{4} \div 1760$; that is, $\frac{4840 \times 4}{1760}$.

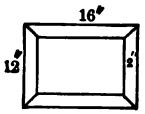
3. Find the distance in miles traveled by a team (*a*) in cutting an acre of grass when the machine cuts a strip 4 ft. 6 in. wide; (*b*) in cutting an acre of wheat when the machine cuts a strip 6 ft. wide.

4. If a team travels 15 miles in a day, (*a*) find the number of acres it can plow when the furrow is 9 inches wide; (*b*) the number of acres of grass it can cut when the mowing machine cuts a strip $4\frac{1}{2}$ ft. wide; (*c*) the number of acres of grain it can cut when the reaper cuts a strip 6 ft. wide.



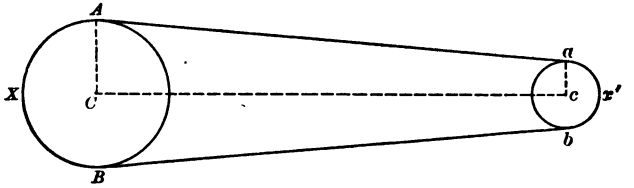
5. An hexagonal mirror *abcdef* is made from a rectangular plate, *ABCD*. (*a*) Find the dimensions of the plate required for a mirror having sides 6 inches long. (*b*) Find the area of the four triangles cut from the plate in making the mirror. (*c*) Find the number of square inches in the mirror.

6. In a picture frame 2 inches wide, (*a*) how many square inches are used, including waste, when the outside dimensions are 16 in. by 12 in.? (*b*) How many square inches are cut off in making the four corners? (*c*) How many square inches are there in the frame?



7. (*a*) What is the area of the largest circular piece of tin that can be cut from a sheet $3\frac{1}{2}$ feet square? (*b*) How many square feet of tin remain? (*c*) What fraction of the original sheet is left?

8. Two wheels are connected by a continuous belt. Find the length of the belt when the radii of the wheels are 11 inches and 24 inches respectively and the centers are 84 inches apart, assuming that the belt touches each wheel for one half of its circumference.



Note that Aa is the hypotenuse of a right triangle whose base is Cc and whose perpendicular is $AC - ac$.

9. To make the top of a vegetable can a piece of tin 7 inches in diameter is required. (*a*) How many square inches are used for the top and the bottom? (*b*) What is the length of the strip used in making the side when $\frac{1}{2}$ inch is required by the seam? (*c*) What is the breadth of the strip required for a can 6 inches high when $\frac{1}{2}$ inch is used for the seam at each base? (*d*) How many square inches of tin are required for a can? (*e*) Find the capacity of the can.

SECTION VIII

EQUATIONS IN BUSINESS

Formulas

A builder is asked what load a certain beam will bear. Turning to his handbook he finds the following formula for the safe load of a rectangular beam supported at both ends and uniformly loaded over the entire span.

$$\text{Safe load} = 2 \times \frac{w \times d^2 \times C}{l}$$

An explanatory note states that w represents the width of the beam in inches, d its depth in inches, and l its length in feet between the points of support. The value of C is given in a table which shows the equivalent in pounds for different woods; yellow pine, oak, spruce, white pine, hemlock, etc.

If the beams are of spruce 3 inches wide, 6 inches deep, and 12 feet long, and the table gives 70 pounds as the safe unit for spruce, these figures are substituted in the foregoing formula, thus:

$$\text{Safe load (in pounds)} = 2 \times \frac{3 \times 36 \times 70}{12}$$

Preparatory Exercises

1. Find the result.

2. What is the safe load when yellow pine is used in a similar case, its unit being 100 pounds?

Equations

The foregoing formula constitutes an equation which consists of two *members* connected by a sign of equality, one of the members containing an *unknown number* whose value is to be determined.

NOTE. — The expression $6 + 5 = 11$ is not an equation; it is called an *identity*.

In these two examples the result is determined by substituting the given numbers and performing the indicated operations. In the following, intermediate steps are required to obtain the result.

3. A builder wishes to ascertain the depth of a yellow pine beam 12 feet long between supports and 3 inches wide that will sustain a load of 1800 pounds uniformly distributed.

He substitutes in formula (*a*) the given numbers, producing the following equation:

$$1800 = \frac{2 \times 3 \times d^2 \times 100}{12}.$$

He first *simplifies* the second member by cancellation, making it $50\,d^2$, which gives the following:

$$1800 = 50\,d^2,$$

or, as it is customary to make the first member the one containing the unknown number:

$$50\,d^2 = 1800.$$

Dividing both members by 50, the equation becomes

$$d^2 = 36.$$

Extracting the square root of both members,

$$d = 6. \quad \textit{Ans.} \quad 6 \text{ inches deep.}$$

Test by substituting 36 for d^2 in the formula, which should give 1800 lb. for the safe load.

Written Exercises

1. How wide should be a yellow pine 12-foot beam, 6 inches deep, to sustain a load of 2400 pounds?

2. What should be the distance between the points of support of a yellow pine beam 3 inches wide and 6 inches deep to enable it to sustain a load of 2400 pounds?

$$2400 = \frac{2 \times 3 \times 36 \times 100}{l}.$$

Simplify the second member, producing the following:

$$2400 = \frac{21600}{l}.$$

Remove the fraction from the second member by multiplying both members by l.

$$2400\,l = 21600.$$

3. Find the equivalent required for C to enable a 12-foot beam, 3 inches wide and 6 inches deep, to sustain a load of 1350 pounds; that is, solve the equation:

$$\frac{2 \times 3 \times 36 \times C}{12} = 1350.$$

4. Find the area of a field in the form of a trapezoid whose parallel sides measure 30 and 50 rods, respectively, and having a perpendicular distance between them of 35 rods.

$$A = \frac{(30 + 50) \times 35}{2}.$$

First combine the numbers in the parenthesis. Then cancel.

5. The area of a trapezoid is 1200 square rods. The parallel sides are, respectively, 25 rods and 35 rods long. Find the perpendicular distance between them.

$$\frac{(25 + 35) \times p}{2} = 1200.$$

Percentage

The general formula for finding a per cent of a number is

$$P = \frac{B \times R}{100},$$

in which B represents the number, R the *rate*, and P the result. The letter B is called the *base* and P the *percentage*.

When the cost of an article is given and the rate of gain or of loss, the gain or the loss is obtained by multiplying the cost by the rate expressed as hundredths.

The difference in the live weight of a cow and the weight of its meat in the butcher shop is about 48%. If the live weight is 950 lb., the loss is 48% of 950 lb.

In applying the formula care must be used to select the proper number for the base. As a rule this is expressed in the problem, but such is not the case in the following:

6. The population of a city was 18,000 in 1915, an increase over 1914 of 12½%. What was the increase?

7. A dealer sold goods for $18,000, which represented a profit of 12½%. What was his profit?

In example 6, the base is the population of 1914; in 7, it is the *cost*, unless it be specified that in this case the profit is 12½% of the receipts.

A person called upon to calculate the interest on $300 for 67 days at 6% could refer to the formula:

$$I = \frac{P \times R \times T}{100}.$$

Replacing the letters in the formula by the given numbers, he obtains the following:

$$I = \frac{\$300 \times 6 \times 67}{100 \times 360}.$$

To find Principal, Rate, Time, etc.

Written Exercises

1. What principal at $3\frac{1}{2}\%$ will yield \$34.23 interest in 2 yr. 8 mo. 18 da.?

PROCESS

Represent the principal by P, the rate in hundredths as $\frac{7}{200}$, and the time (978 days) in years as $\frac{978}{360}$. The combined product of the foregoing represents the interest, \$34.23.

$$\frac{P \times 7 \times 978}{200 \times 360} = 34.23$$

Clear this of fractions by multiplying both members by 200×360, which gives

$$P \times 7 \times 978 = 34.23 \times 200 \times 360$$

Cancel the decimal point in 34.23 and the two ciphers in 200, which gives

$$P \times 7 \times 978 = 3423 \times 2 \times 360$$

Indicate the division of both members by 7×978, which gives

$$P = \frac{3423 \times 2 \times 360}{7 \times 978}$$

Cancel and find the result.

Test by calculating the interest on the principal thus found.

2. Find the interest on \$360 for 2 yr. 8 mo. 18 da. at $3\frac{1}{2}\%$.

3. At what rate will \$540 produce \$88.83 interest in 4 yr. 8 mo. 12 da.?

PROCESS

Represent the rate by R, change 4 yr. 8 mo. 12 da. to $\frac{1692}{360}$ yr., which gives

$$\frac{540 \times R \times 1692}{100 \times 360} = 88.83$$

from which

$$R = \frac{88.83 \times 100 \times 360}{540 \times 1692}$$

Reject the decimal and cancel 100.

4. Find the amount of \$540 for 4 yr. 8 mo. 12 da. at $3\frac{1}{2}\%$.

5. At what rate will \$360 amount to \$419.22 in 4 yr. 8 mo. 12 da. ?

PROCESS

First find the interest, which is \$419.22 − \$360. Substitute the result in the formula, as in the preceding example.

6. In what time will \$240 at $3\frac{1}{2}\%$ yield \$38.99 interest?

PROCESS

Represent the time (in years) by T.

$$\frac{240 \times 7 \times T}{200} = 38.99$$

Indicate the value of T.

$$T = \frac{38.99 \times 200}{240 \times 7}$$

Cancel the decimal point, etc. Change the fraction of a year in the result to months and a fraction, and the latter to days.

7. In what time will \$240 at $3\frac{1}{2}\%$ amount to \$278.99? First, find the interest (\$278.99 − \$240).

8. What principal will amount to \$563.82 in 3 yr. 10 mo. 17 da. at $4\frac{1}{2}\%$?

PROCESS

Indicate the interest on P at $4\frac{1}{2}\%$ for 3 yr. 10 mo. 17 da. (1397 da.).

$$\frac{P \times 9 \times 1397}{200 \times 360} = \text{Interest}$$

Simplify the first member by cancellation, etc., making the equation

$$\frac{1397 \times P}{8000} = I$$

Find the amount by adding the principal

$$P + \frac{1397\,P}{8000} = A = 563.82$$

$$\text{That is, } \frac{8000\,P}{8000} + \frac{1397\,P}{8000} = 563.82$$

Combine the two terms in the first member

$$\frac{9397\,P}{8000} = 563.82$$

$$P = \frac{563.82 \times 8000}{9397}$$

9. Find principal, rate, time, etc., as required.

a. Principal, \$420; rate, 5%; time, 4 yr. Interest?
b. Principal, \$350; rate, 4%; time, 3 yr. Amount?
c. Principal, \$480; rate, 6%; interest, \$86.40. Time?
d. Principal, \$250; time, $2\frac{1}{2}$ yr.; interest, \$37.50. Rate?

Equations in General

Preparatory Exercises

a. What number added to 54 gives 72?
b. What number subtracted from 54 leaves 18?
c. What number multiplied by 72 gives 54?
d. What number divided by 4 gives 18?

One method of solving such problems is to write them as follows:

a. $\begin{array}{r} 54 \\ +\,? \\ \hline 72 \end{array}$ *b.* $\begin{array}{r} ? \\ -\,54 \\ \hline 18 \end{array}$ *c.* $\begin{array}{r} 72 \\ \times\,? \\ \hline 54 \end{array}$ *d.* $\frac{?}{4} = 18$ or $4\overline{)\,?}$ $\quad 18$

and then to determine the number that will produce the result.

A general method for the solution of problems of this kind is to write each as an equation, thus:

a. $54 + x = 72$ *b.* $x - 54 = 18$ *c.* $72x = 54$ *d.* $\frac{x}{4} = 18$

and then to solve the equation.

Only + and − signs should be used in equations, the product of an ordinary number by an unknown number being indicated by writing them together ($72x$), and division being indicated by writing the divisor as the denominator of a fraction of which the dividend is the numerator. The product of two ordinary numbers is generally obtained before the equation is written.

A number written before a letter without a sign connecting them, is called a *coefficient* of the letter. Thus, 72 in (*c*) is the coefficient of x. When no coefficient is expressed, the coefficient is 1.

The solution of an equation consists in finding the value of the unknown number.

Sight Exercises

1. Supply the missing number:

a. $? + 3 = 19$	*e.* $3 \times ? = 24$	*i.* $? + 2 = 15$
b. $? - 9 = 18$	*f.* $? \times 5 = 40$	*j.* $6 + ? = 12$
c. $? - 7 = 25$	*g.* $9 \times ? = 81$	*k.* $? + 4 = 10$
d. $3 + ? = 90$	*h.* $? \times 4 = 68$	*l.* $5 + ? = 15$

Collecting Terms

To solve the equation

$$6x + x - 2x - 4x + 3x = 68 - 80 + 60$$

it is necessary to combine the five terms of the first member into a single term, and also the three terms of the second member.

The five terms of the first member consist of three *positive* terms $+ 6x$, $+ x$, and $+ 3x$, whose sum is $+ 10x$. The sum of the two *negative* terms, $- 2x$ and $- 4x$, is $- 6x$. The first member is, therefore, equivalent to $10x - 6x$, which is combined into $4x$.

The two positive terms of the second member, $+ 68$ and $+ 60$, are combined into 128, the second member becoming $128 - 80$, which is combined into 48.

The simplified equation, $4x = 48$, is solved by dividing both members by 4, the coefficient of x, which gives $x = 12$.

To test the result, replace x in the *original* equation by 12, which gives: $72 + 12 - 24 - 48 + 36 = 68 - 80 + 60$.

Collect the terms in each member.

Sight Exercises

Give value of x:

a. $x + 2x = 60 - 30$	*b.* $x + 2x + 3x = 30 + 18$
c. $2x + 3x = 20 + 10$	*d.* $x - 2x + 3x = 30 - 10$
e. $3x - x = 40 - 10$	*f.* $2x - 3x + 5x = 20 + 28$

Written Problems

1. A boy paid $220 for a horse, a buggy, and a set of harness. The buggy cost 4 times as much as the harness and the horse cost $1\frac{1}{2}$ times as much as the buggy. Find the cost of each.

PROCESS

Let x = cost of the harness
Then $4x$ = cost of buggy
And $6x$ = cost of horse

$x + 4x + 6x = 220$
Collect terms $11x = 220$
Divide by 11 $x = 20$, cost of harness in dollars
$4x = 80$, cost of buggy in dollars
$6x = 120$, cost of horse in dollars
Test: Sum $= 220$ *Ans.* $20, $80, $120

In equations, only abstract numbers are employed, x in this problem denoting the *number* of dollars. In writing the answers, the dollar sign is prefixed.

2. In four flocks there are 360 sheep. The first numbers twice as many as the second, and the second as many as the third and the fourth together. If these two contain the same number, how many are there in each flock?

Let x = number in the fourth flock
Then x = number in the third flock
$2x$ = number in the second flock
And $4x$ = number in the first flock
$x + x + 2x + 4x = 360$

Collect terms.

3. Divide 78 marbles between two boys so that one will have 5 times as many as the other.

4. A man spends \$3.04 for coffee at 16 cents per pound and tea at 60 cents per pound, buying the same number of pounds of each. How many pounds of each does he buy?

Let x = number of pounds of each

$16x + 60x = 304$

As the cost of the articles is given in cents, the sum spent, \$3.04, is written as cents.

5. A drover bought sheep, cows, and horses, an equal number of each, for \$4440, paying 6 times as much for a cow as for a sheep, and 5 times as much for a horse as for a cow. How many of each did he buy?

6. A farmer obtained from 60 acres in wheat and 40 acres in corn, a total yield of 3600 bushels of grain. The corn field produced three times as much grain per acre as the wheat field. How many bushels were obtained from each?

7. Mr. Adams has 3 farms containing together 360 acres. The first contains $1\frac{1}{8}$ times as many acres as the second, and the second contains $1\frac{1}{2}$ times as many as the third. How many acres are there in each?

Let $2x$ = number of acres in the third

8. A boy buys three times as many 1¢ stamps as he buys of 2's, and twice as many 2's as he buys of 5's. How many of each does he buy if he pays 90 cents for the lot?

Let x = number of 5-cent stamps

Then $2x$ = number of 2-cent stamps

And $6x$ = number of 1-cent stamps

$5x$ = cost of 5-cent stamps

$4x$ = cost of 2-cent stamps

$6x$ = cost of 1-cent stamps

$5x + 4x + 6x = 90$

Transposing Terms

Sight Exercises

1. Give value of x:

a. $x+20=40$	*b.* $x+30=70$	*c.* $x+40=90$
d. $x-20=40$	*e.* $x-30=70$	*f.* $x-40=90$

2. Give value of $2x$:

a. $2x+2=10$	*b.* $2x+2=16$	*c.* $2x+4=20$
d. $2x-2=10$	*e.* $2x-2=16$	*f.* $2x-4=20$

3. Give value of x:

a. $2x+2=10$	*b.* $2x+2=16$	*c.* $2x+4=20$
d. $2x-2=10$	*e.* $2x-2=16$	*f.* $2x-4=20$
g. $3x+2=17$	*h.* $3x-2=19$	*i.* $4x-4=20$

To find the value of x in the equation $x + 20 = 40$, the first member must be diminished by 20; and to preserve the equality, 20 must also be subtracted from the second member.

$$x + 20 - 20 = 40 - 20$$

Combining terms in each member

$$x = 20$$

To find the value of x in the equation $x - 20 = 40$, the first member must be increased by 20, and to preserve the equality, 20 must be added to the second member.

$$x - 20 + 20 = 40 + 20$$

Combining terms in each member

$$x = 60$$

In practice, however, it is not customary to take the unnecessary steps of increasing or decreasing both members of an equation by writing the members on both sides.

The equations, $x + 20 = 40$ $\qquad$ $x - 20 = 40$
are changed to $x = 40 - 20$ $\qquad$ $x = 40 + 20$

by transposing 20 to the second member with a change of sign.

Written Exercises

1. Find the value of x in the following equations:

(a) $160 - 3x = 250 - 12x$

PROCESS

Given,	$160 - 3x = 250 - 12x$
Transpose,	$-3x + 12x = 250 - 160$
Collect terms,	$9x = 90$
Divide by the coefficient of x,	$x = 10$ *Ans.*

TEST

Substitute -30 in the first member for $-3x$, and -120 in the second member for $-12x$. This gives $160 - 130 = 250 - 120$, which becomes $130 = 130$.

(b) $3x + 240 - 5x - 460 + 4x - 310$

PROCESS

Given,	$3x + 240 - 5x = 460 + 4x - 310$
Transpose,	$3x - 5x - 4x = 460 - 310 - 240$
Collect terms,	$-6x = -90$
Change signs,	$6x = 90$
Divide by coefficient of x,	$x = 15$ *Ans.*

TEST

$$(3 \times 15) + 240 - (5 \times 15) = 460 + (4 \times 15) - 310$$
$$45 + 240 - 75 = 460 + 60 - 310$$
$$285 - 75 = 520 - 310$$
$$210 = 210$$

NOTE. — In the equation $-6x = -90$, the negative signs can be made positive by transposing $-6x$ to the second member and -90 to the first member, thus: $90 = 6x$, which becomes $15 = x$, by dividing both members by 6; but it is customary to change the negative sign of each member.

2. Solve the following equations. Test each result:

a. $19x-22=90+3x$ *e.* $9x-15=24-6x+18-4x$
b. $17x+18=94-2x$ *f.* $6x-42+5x=34-3x-5x$
c. $96+4x=16x-48$ *g.* $57-6x+9=3x+8-x+2$
d. $14x+63=198-x$ *h.* $24-3x-6x=33-4x-64$

Sight Problems

1. When 7 is subtracted from 5 times a number the remainder is 28. What is the number?

2. What number increased by twice itself and by 13 will be 43?

3. Henry has 40 marbles, which is 12 fewer than twice as many as William has. How many has William?

4. Mr. Smith has 45 head of stock. He has 10 more cows and 20 more sheep then he has horses. How many has he of each?

5. When 10 is added to 3 times a number the sum is 100. Find the number.

6. In a class of 40 pupils the girls outnumber the boys by 2. How many boys are in the class?

7. A team of horses and a wagon cost together \$750. What is the cost of each horse if it is \$150 more than that of the wagon?

8. The length of a field exceeds its width by 10 rods. Its perimeter is 140 rods. Give the dimensions.

Sight Exercises

1. Solve the following equations:

a. $4x+9=53$ *b.* $5x=16-3x$ *c.* $5x-17=3x+11$
d. $3x-4=23$ *e.* $4x=28+2x$ *f.* $7x+20=50-3x$
g. $6x+2=38$ *h.* $7x=36-2x$ *i.* $2x-15=18-9x$

Written Problems

1. The difference of two numbers is 24 and their sum is 86. Find the numbers.

PROCESS

Let	$x =$ the smaller number
Then	$x + 24 =$ the larger
	$x + x + 24 = 86$
Transpose	$x + x = 86 - 24$
Combine	$2x = 62$
Divide	$x = 31$, the smaller number
	$x + 24 = 55$, the larger number
Test.	$31 + 55 = 86$

In testing the accuracy of a result, be careful to select the proper condition. Since 55, the larger number, is obtained by increasing the smaller number, 31, by 24, their difference, which is the first condition in the problem, the correctness of the answer 31 is not determined by subtracting it from 55. Test by the other condition, which gives 86 as the sum. Since 31 + 55 gives this result, the answers are correct.

2. A girl planted tulips, hyacinths, and crocuses; 20 more hyacinths than tulips, and 25 more crocuses than hyacinths. How many of each were there, if she planted 95 in all?

PROCESS

Let	$x =$ number of tulips	
Then	$x + 20 =$ number of hyacinths	
And	$x + 45 =$ number of crocuses	
	$3x + 65 = 95$	etc.

After the pupil has had some practice, he can write the equation

$$x + x + 20 + x + 20 + 25 = 95$$

without the preliminary statement.

In writing the latter, he should employ only as many words as he finds necessary, regardless of the number given in the sample statements.

3. Find two numbers whose sum is 115 and whose difference is 43.

4. A man has two fields, containing together 73 acres, one of which contains 19 acres more than the other. How many acres are there in each?

5. The length of a rectangle is 44 yards greater than its width, and the perimeter is 240 yards. (*a*) Find the dimensions. (*b*) Find the area.

$$x + x + 44 + x + x + 44 = 240.$$

6. When 3 times a number is subtracted from 142, the result is the same as the subtraction of 18 from 5 times the number. Find the number.

Clearing of Fractions

Preparatory Exercises

Give the value of x, when

a. $\frac{1}{4}$ of x is 12	*d*. $\frac{3}{4}$ of x is 12	*g*. $1\frac{1}{2}$ times x is 12
b. $\frac{1}{5}$ of x is 10	*e*. $\frac{2}{5}$ of x is 10	*h*. $3\frac{1}{3}$ times x is 10
c. $\frac{1}{7}$ of x is 21	*f*. $\frac{3}{7}$ of x is 21	*i*. $1\frac{2}{5}$ times x is 21

The expression $\frac{1}{4}$ of x is written $\frac{x}{4}$; $\frac{2}{5}$ of x is written $\frac{2x}{5}$; $3\frac{1}{3}$ times x is written $\frac{10x}{3}$, a mixed number being generally written as an improper fraction.

Written Exercises

1. Find the value of x in the following equation:

$$3x - 60 + \frac{2x}{3} = 58 - \frac{5x}{4}.$$

In order to obtain an equation without fractions multiply both members by the L. C. M. of the denominators of the fractions.

PROCESS

Given	$3x - 60 + \frac{2x}{3} = 58 - \frac{5x}{4}$
Multiply each term by 12	
	$36x - 720 + 8x = 696 - 15x$
Transpose	$36x + 8x + 15x = 696 + 720$
Combine	$59x = 1416$
Divide	$x = 24$ *Ans.*
Test.	$72 - 60 + 16 = 58 - 30$; or $28 = 28$

2. Solve the following equations. Test each result.

a. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 91$

b. $x + \frac{x}{3} + \frac{x}{5} = 92$

c. $2x + \frac{x}{2} - \frac{x}{3} = 26$

d. $\frac{2x}{3} + \frac{x}{4} - \frac{x}{5} = 43$

e. $\frac{x}{3} - \frac{x}{4} + \frac{x}{5} = 34$

f. $\frac{x}{3} + 19 = \frac{x}{4} + 20$

g. $\frac{2x}{5} + \frac{2x}{3} = x + 25$

h. $\frac{x}{3} + \frac{x}{4} = \frac{5x}{8} - \frac{11}{12}$

i. $3x - 7 = \frac{5x}{3} + \frac{7}{3}$

j. $\frac{x}{3} + \frac{x}{4} = \frac{2x}{5} + 11$

Written Problems

1. What number increased by 50 % and $33\frac{1}{3}$ % of itself will equal 66?

$$x + \frac{x}{2} + \frac{x}{3} = 66.$$

2. The difference between 75 % and $66\frac{2}{3}$ % of the same number is 13. Find the number.

3. A certain number is increased by 56 and the sum divided by 12, giving 10 for the quotient. What is the number?

$$\frac{x + 56}{10} = 12.$$

4. After selling $\frac{1}{2}$ and $\frac{1}{3}$ of his land, Mr. Yates has still 100 acres. How many acres had he originally?

$$x - \frac{x}{2} - \frac{x}{3} = 100.$$

5. After a discount of 50 % of the list price and $33\frac{1}{3}$ % of the remainder, an article cost \$100. What is the list price?

$$x - \frac{x}{2} - \frac{x}{3} = 100.$$

6. My crop of wheat this year is 561 bushels, which is $37\frac{1}{2}$ % larger than last year's crop. How many bushels did I raise last year?

7. A man leaves \$7500 to his widow, two sons, and three daughters, each son to have twice as much as a daughter, and the widow \$500 more than all the children together. What is the share of each?

Let x = the share of each daughter
Then $2x$ = the share of each son
$3x$ = share of three daughters
$4x$ = share of two sons
$7x + 500$ = widow's share

8. Divide 57 stamps among 3 girls in such a manner that the second will receive $1\frac{1}{2}$ times as many as the first, and the third will receive $1\frac{1}{2}$ times as many as the second.

9. The sum of two numbers is 75, one being $1\frac{1}{2}$ times the other. Find the numbers.

10. The perpendicular distance between the parallel sides of a trapezoid is 60 rods. One of the parallel sides is 50 rods. Find the other, when the area is 3300 square rods.

$$\frac{(50 + x) \times 60}{2} = 3300.$$

Removing Parentheses

A parenthesis is used to combine two or more terms into a single compound term. The expression $60 - (30 + 10)$ is considered as composed of two terms, the first of which, 60, is to be diminished by the second, the latter being a compound expression indicating the difference between 30 and 10. The two numbers written within the parenthesis are considered positive, the $-$ sign preceding the parenthesis indicating the operation to be performed.

When a parenthesis preceded by a $-$ sign is removed, the signs of the contained numbers are changed. Thus, the foregoing expression is written $60 - 30 - 10$. The expression $60 - (30 - 10)$ is written $60 - 30 + 10$ when the parenthesis is removed.

A number immediately preceding, or following, a parenthesis, and not connected with it by any sign, indicates that it is a multiplier of the enclosed expression. Thus, $60 - 2\ (30 - 10)$ means that 60 is to be diminished by twice the difference between 30 and 10. The multiplier, 2, may be placed within the parenthesis by performing the indicated multiplication, which gives $60 - (60 - 20)$, and the parenthesis may then be removed, which gives $60 - 60 + 20$.

Written Exercises

1. Find the value of x in the equation :

$$x - 3(x - 10) = 4x - 42.$$

PROCESS

Given	$x - 3(x - 10) = 4x - 42$
Remove the parenthesis	$x - 3x + 30 = 4x - 42$
Transpose	$x - 3x - 4x = 42 - 30$
Combine	$-6x = -72$
Change signs and divide	$x = 12$, *Ans.*

Another method is to transpose the negative compound term, making it positive

$$x = 4x - 42 + 3(x - 10).$$

In making such transposition change only the sign *prefixed* to the compound term.

2. Solve the following equations. Test results.

a. $4(x - 3) + 7(x - 1) = 5x - 1.$

b. $4(x + 3) - 7(x - 3) = 2(x + 4).$

c. $8(x + 2) - 9(x - 2) = 3x + 2.$

d. $6(10 - x) - 9(8 - x) = x + 1.$

e. $48 - 3(4x - 5) = 27 - 3(2x - 4).$

Fractional Terms

1. Find the value of x in the following equation :

$$\frac{2x - 4}{9} - 1 = \frac{x + 1}{12}.$$

Multiply by 36 $\quad \frac{36(2x - 4)}{9} - 36 = \frac{36(x + 1)}{12}$ $\quad (a)$

Simplify $\quad 4(2x - 4) - 36 = 3(x + 1)$ $\quad (b)$

Remove parentheses $\quad 8x - 16 - 36 = 3x + 3$ $\quad (c)$ etc.

The illustration shows the successive steps in the process. The pupil should omit steps (a) and (b) unless he finds them necessary.

2. Solve the following equation :

$$6 - \frac{2x-1}{9} = 14 - \frac{5x-4}{6}.$$

	PROCESS
Given	$6 - \frac{2x-1}{9} = 14 - \frac{5x-4}{6}$
Transpose the fractions	$6 + \frac{5x-4}{5} = 14 + \frac{2x-1}{9}$
Transpose 6 and combine	$\frac{5x-4}{6} = 8 + \frac{2x-1}{9}$
Multiply by 18	$3(5x-4) = 144 + 2(2x-1)$
	etc.

By transposing a negative fractional term, thus changing it to a positive one, the beginner sometimes avoids mistakes. The transposition of 6 and its combination with 14 reduces the number of terms.

3. Solve the following equations :

a. $\frac{x-9}{12} - \frac{x-6}{8} + 1 = \frac{x-10}{4}.$

b. $\frac{x+1}{5} - \frac{2-x}{7} = \frac{x-3}{2}.$

c. $\frac{2x+3}{9} - \frac{x-3}{3} = 9 - \frac{5x-6}{6}.$

d. $\frac{x+1}{6} = \frac{x-1}{4} - \frac{20-x}{3}.$

e. $\frac{3x+2}{7} - \frac{x+3}{2} + 5 = \frac{3x-21}{4}.$

f. $\frac{3x+1}{4} - \frac{4x+1}{3} = \frac{x-23}{6}.$

Written Problems

1. In a factory 36 men and 66 boys are employed, each man receiving \$2 per day more than a boy. What is the pay of each if the daily pay of all is \$225?

$$36(x+2)+66x=225.$$

2. A fraction is equal to $\frac{3}{4}$, and the difference between its numerator and its denominator is 5. What is the fraction?

$$\frac{x}{x+5}=\frac{3}{4}.$$

When each member consists of a single fraction, the equation is cleared of fractions by multiplying the numerator of the fraction in the first number by the denominator of the fraction in the second, and the numerator of the fraction in the second by the denominator of the fraction in the first.

$$4x=3(x+5).$$

3. If 3 be added to both terms of a fraction whose value is $\frac{3}{4}$, the value will become $\frac{4}{5}$. What is the fraction?

Representing the fraction by $\frac{3x}{4x}$,

$$\frac{3x+3}{4x+3}=\frac{4}{5}.$$

4. A train going 15 miles per hour faster than a second train travels 180 miles while the other travels 120 miles. At what rate does each travel?

$$\frac{180}{x+15}=\frac{120}{x}.$$

5. What fraction equal to $\frac{5}{7}$ will become equal to $\frac{2}{3}$ when 10 is subtracted from both terms?

6. What per cent of $18\frac{3}{4}$ is $16\frac{2}{3}$?

$$\frac{x}{100}\text{ of }18\tfrac{3}{4}=16\tfrac{2}{3}.$$

7. A man bought a number of bicycles at \$14 each. He sold $\frac{2}{3}$ of them at \$19 each, 2 of them at \$18 each, and the remainder at \$16 each, realizing a profit of \$100. How many did he buy?

Representing by x the number bought, $\frac{2x}{3}$ is the number sold at \$19 each. There are then $\frac{x}{3}$ left, of which 2 are sold at \$18 each, leaving $\frac{x}{3} - 2$, which are sold at \$16 each.

The profit in dollars on the first lot is 5 times $\frac{2x}{3}$, or $\frac{10x}{3}$; on the second lot, 8; on the third lot, 2 times $\left(\frac{x}{3} - 2\right)$, or $\frac{2x}{3} - 4$.

The whole profit, in dollars, is $\frac{10x}{3} + 8 + \frac{2x}{3} - 4$, which is equal to 100.

8. A dealer buys a certain number of bicycles at \$13 each. He sells $\frac{2}{3}$ of them at \$18 each, 3 at \$17 each, and the remainder at \$15 each, making a total profit of \$78. How many did he buy?

9. What per cent of $16\frac{2}{3}$ is $18\frac{3}{4}$?

10. By what fraction of itself must $16\frac{2}{3}$ be increased to become $18\frac{3}{4}$?

$$16\tfrac{2}{3} + 16\tfrac{2}{3}x = 18\tfrac{3}{4}.$$

11. By what fraction of itself must $18\frac{3}{4}$ be diminished to become $16\frac{2}{3}$?

12. The perimeter of a rectangle is 48 rods. Find the area when one side is 4 rods longer than the adjacent side.

Let x = length of short side.
Find the area after dimensions are determined.

13. A and B purchase together 350 tons of hay at \$12 per ton. If A takes 60 tons more than B, what should each pay?

14. A man wishing to divide a number of stamps among some boys finds, if he gives 5 to each, he will have 10 over, and that he requires 5 more in order to give 6 to each. How many stamps has he?

Let x = number of boys.

$$6x - 5 = 5x + 10.$$

After ascertaining the number of boys, the number of stamps can be found.

15. I had \$4 in my pocket. I spent 60 cents for a book and I bought two pairs of gloves, one pair costing 40 cents more than the other. I then had \$1.80 left. What was the price of each pair of gloves?

16. By purchasing cloth at \$2.40 per yard, I have 40 cents left. If I had bought cloth at \$2.80 per yard, I should have been 70 cents short. How many yards did I buy?

17. How many pounds of coffee costing 24¢ per pound can be mixed with coffee costing 18¢ per pound to make 150 pounds costing 22¢ per pound? Solve and test.

PROCESS

Let x = quantity of 24¢ coffee

Then $150 - x$ = quantity of 18¢ coffee

$24x$ = cost of first (in cents)

$18(150 - x)$ = cost of second (in cents)

and 150×22 = cost of mixture (in cents)

$$24x + 18(150 - x) = 3300$$

$$24x + 2700 - 18x = 3300$$

etc.

The richness of milk depends upon the percentage of butter fat it contains. Below 3% its sale in cities is generally forbidden as not being sufficiently nutritious for young children. Above 18% it is considered cream.

18. A creamery has an order for 150 gallons of 4% milk. (*a*) There is on hand a lot of milk testing 5% and another lot testing $3\frac{1}{2}$%. How many gallons of each can be used in a mixture that will fill the order?

Taking x as the quantity of 5% milk and $150 - x$ as the quantity of $3\frac{1}{2}$% milk, the equation becomes,

$$\frac{5x}{100} + \frac{3\frac{1}{2}(150 - x)}{100} = \frac{4(150)}{100}.$$

Clear the equation of the denominator 100 by rejecting it from each term. Solve the equation. Test.

(*b*) How could the same order be filled with skim milk and 6% milk?

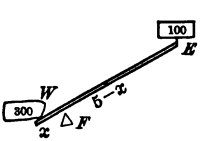
$$\frac{5x}{100} + \frac{0(150 - x)}{100} = \frac{4(150)}{100}.$$

Since skim milk has no butter fat, its per cent is 0; hence the second term of the first member of the foregoing equation disappears.

19. If there is a call for 75 pounds of milk containing 3.5% of fat, and milk containing 6% is to be used, how much skim milk can be added?

20. A boy weighing 100 pounds is sitting on one end of a 5-ft. crowbar, the other end of which is under a stone. (*a*) How far from the stone must a support be placed in order that his weight will exert a lifting force of 300 pounds upon the stone?

In a lever, the energy (E) multiplied by its distance from the fulcrum (F) is equal to the work (W) multiplied by its distance from the fulcrum.



Representing the distance FW by x, EF will be $5 - x$. Multiplying the former by 300 and the latter by 100, the following equation results:

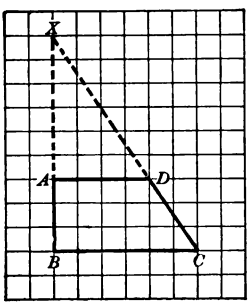
$$300x = 100(5 - x).$$

(*b*) Assuming that the crowbar extends 6 inches under the stone, how far from the edge of the stone must the fulcrum be placed?

$$300\,x = 100(4\tfrac{1}{2} - x).$$

21. A dairyman wishes to deliver 100 pounds of cream testing 18%. How many pounds of 5% milk and of 25% cream can he use?

22. A portion of a field in the form of a right triangle was cut off by a line parallel to the base, making a trapezoidal field with parallel sides measuring, respectively, 48 rods and 72 rods, the perpendicular distance between them being 36 rods. What was the length of the perpendicular of the original triangle?



This may be worked experimentally on cross-ruled paper by drawing $ABCD$ to scale, making BC 6 spaces, AD 4 spaces, and AB 3 spaces. Extend CD and BA until they meet?

PROCESS

The line AD, parallel to BC, makes two *similar* triangles, AXD and BXC. In triangles of this kind the homologous (corresponding) sides are proportional; that is,

$$XA : XB :: AD : BC$$

Representing XB by x, XA will be $x - 36$;

therefore $x - 36 : x :: 48 : 72$

or, $:: 2 : 3$ (Canceling)

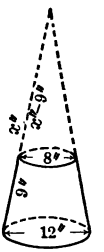
Making the product of the extremes equal to the product of the means, the following equation results:

$$3\,x - 108 = 2\,x$$

or, $x = 108$ *Ans.* 108 rods

Test. Compare this result with the one obtained by the drawing.

23. The upper portion of a cone was cut off, leaving a frustum with bases 8″ and 12″ in diameter, respectively, and a slant height of 9″. What was the slant height of the original cone?

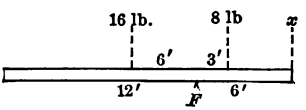


Find the value of x in the following proportion:

$$x : 12 :: x - 9 : 8.$$

24. An 18-foot plank weighing 24 pounds rests on a fulcrum placed 12 feet from one end. (*a*) What weight must be placed on the other to balance the plank?

The weight of each portion is assumed to be concentrated at its center, that of the larger part (16 lb.) being located 6 ft. from the fulcrum, and that of the smaller part (8 lb.) 3 ft. from the fulcrum.



The balancing weight (x) is 6 ft. from the fulcrum.

Using a dot to denote multiplication,

$$x \cdot 6 + 8 \cdot 3 = 16 \cdot 6,$$

that is,

$$6x + 24 = 96.$$

25. A boy weighing 80 pounds is sitting at one end of an 18-ft. plank that weighs 24 pounds. If he is 12 feet from the fulcrum, how heavy a man can he balance at the other end of the plank?

80 (lb.) × 12 (ft.) + 16 (lb.) × 6 (ft.) must balance x (lb.) × 6 (ft.) + 8 (lb.) × 3 (ft.).

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TABLES

LINEAR MEASURE

12	inches (in.)	= 1 foot	ft.
3	feet	= 1 yard	yd.
5½	yards, or 16½ feet	= 1 rod	rd.
40	rods	= 1 furlong	fur.
320	rods	= 1 mile	mi.

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

A *hand*, used in measuring the height of horses, = 4 in. A *knot*, used in measuring distances at sea, = 1.15 mi. A *fathom*, used in measuring the depth of the sea, = 6 ft.

SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot	sq. ft.
9	square feet	= 1 square yard	sq. yd.
30¼	sq. yd., or 272¼ sq. ft.	= 1 square rod	sq. rd.
160	square rods	= 1 acre	A.
640	acres	= 1 square mile	sq. mi.

1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.

A Section of land is a square mile.

Roofing, flooring, and slating are often estimated by the *square*, which contains 100 square feet.

SURVEYORS' MEASURE

In measuring land, surveyors use a chain (ch.) which contains 100 links (l.) and is 4 rods long. Since the chain is 4 rods long, a square chain contains 16 sq. rd., and 10 sq. ch. = 160 sq. rd., or 1 acre.

CUBIC MEASURE

1728	cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27	cubic feet	= 1 cubic yard	cu. yd.
128	cubic feet	= 1 cord	cd.
16	cubic feet	= 1 cord ft.	cd. ft.
8	cord feet	= 1 cord	cd.

NOTE. — In computing the contents of an enclosing wall, masons and brick-layers regard it as one straight wall whose length is the distance around it on the outside. Corners are thus measured twice.

A *perch* of stone or masonry is 16½ ft. long, 1½ ft. thick, and 1 ft. high, and contains 24¾ cu. ft.

qt.
pk.
bu.

2 quarts = 1 gallon . . .

The *standard gallon* contains 231 cubic inches.
The *standard bushel* contains 2150.42 cubic inches.

The capacity of cisterns, reservoirs, etc., is often expressed in barrels (bbl.) of 31½ gallons each, or in hogsheads (hhd.) of 63 gallons each. In commerce, these vary in size.

AVOIRDUPOIS WEIGHT

16 ounces (oz.) . . .	= 1 pound	lb.
100 pounds	= 1 hundredweight . .	cwt.
2000 pounds	= 1 ton	T.

One pound Avoirdupois = 7000 grains.

The *long ton* of 2240 pounds is used in the United States Custom Houses and in weighing coal and iron at the mines.

STANDARD WEIGHTS

1 bushel of wheat . . .	= 60 lb.	1 bushel of potatoes. .	= 60 lb.
1 bushel of corn . . .	= 56 lb.	1 barrel of flour . . .	= 196 lb.
1 bushel of oats . . .	= 32 lb.	1 barrel of pork . . .	= 200 lb.
1 bushel of barley . . .	= 48 lb.	1 keg of nails	= 100 lb.

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight . .	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

One pound Troy = 5760 grains.

APOTHECARIES' WEIGHT

60 grains (gr.)	= 1 dram . . .	dr., or ℥.
8 drams	= 1 ounce . . .	oz., or ℥.
12 ounces	= 1 pound . . .	lb., or ℔.

One pound Apothecaries' weight = 5760 grains.

BRITISH OR STERLING MONEY

4 farthings	= 1 penny	*d.*
12 pence	= 1 shilling	*s.*
20 shillings	= 1 pound	£.
5 shillings	= 1 crown.	

The value of £1 is $4.8665 in United States gold coin.

The unit of French money is 1 franc, which is 19.3 cents. The unit of German money is 1 mark, which is 23.8 cents.